Algebra

Using Symbols

EXERCISE 8A

1. Find the answers to the following statements.
(a) The sum of 12 and 4.
(b) The product of 12 and 4.
(c) The difference between 12 and 4.
(d) 12 divided by 4.
(e) 8 more than 20.
(f) 8 less than 20.
(g) 8 less than half of 20.
(h) 8 more than half of 20.
(i) Twice 8.
(j) 6 more than twice 8.
(k) Half the sum of 8 and 6.
(l) 6 more than half the difference between 8 and 20.
(m) The square of 5.
(n) The square root of 64.
(o) The square root of half of 18.
(p) The sum of 20 and half of 8.
(q) Three times the sum of 4 and 5.
(r) The square root of the sum of 12 and 4.

2. Match the worded statements in question 1 with the mathematical statements below.

A $\frac{1}{2}(8 + 6)$	**B** 2×8	**C** $20 - 8$
D $12/4$	**E** $\frac{1}{2} \times 20 + 8$	**F** $3(4 + 5)$
G $12 + 4$	**H** $\sqrt{64}$	**I** $\frac{1}{2} \times 20 - 8$
J $20 + 8$	**K** $\frac{1}{2}(20 - 8) + 6$	**L** $12 - 4$
M $\sqrt{\frac{1}{2} \times 18}$	**N** $20 + \frac{1}{2} \times 8$	**O** 5^2
P $\sqrt{12 + 4}$	**Q** 12×4	**R** $2 \times 8 + 6$

3. Match the following worded statements with the mathematical statements.

(a) The sum of a and b.
(b) The product of a and b.
(c) The difference between a and b.
(d) a divided by b.
(e) 3 more than a.
(f) 3 less than a.
(g) a divided by 3.
(h) a multiplied by 3.
(i) Twice the sum of a and b.
(j) b divided by a.
(k) a squared.
(l) The square root of a.
(m) The square of the sum of a and b.
(n) The square root of the sum of a and b.

A $a - 3$
B $2(a + b)$
C a/b
D $\sqrt{a}$
E b/a
F $a + b$
G $a/3$
H $(a + b)^2$
I $b - a$
J ab
K $\sqrt{a + b}$
L $a + 3$
M $3a$
N a^2

Equations and Formulae

EXERCISE 8B

1. Match the following worded equations with the mathematical equations below.

(a) a is equal to 3 more than b.
(b) a is equal to 3 less than b.
(c) a is equal to the 3 times b.
(d) a is equal to one third of b.
(e) a is equal to 3.
(f) a is equal the sum of b and c.
(g) a is equal to the square of c.
(h) a is equal to the sum of b, c and 3
(i) a is equal to twice the sum of b and c.

A $a = c^2$
B $a = 3b$
C $a = b + c$
D $a = 2(b + c)$
E $a = b + 3$
F $a = 3$
G $a = b - 3$
H $a = b/3$
I $a = b + c + 3$

2. Write the following statements as mathematical equations.

Examples

1. F is equal to the sum of G and H.

$$F = G + H$$

2. M is equal to the product of P and Q.

$$M = PQ$$

Remember: There is no need to include the '×' sign in algebra.

(a) y is equal to the sum of x and w.
(b) K is equal to the product of L and M.
(c) p is equal to 4 more than q.
(d) D is equal to F subtracted from J.
(e) B is equal to the product of A^2 and Z.
(f) Y is equal to the square of the sum of T and U.
(g) e is equal to 7 less than d.
(h) L is equal to 5 more than the product of P and Q.
(i) h is equal to j divided by k.

3. Write the following statements as mathematical equations.

Example

Force (F) is equal to the product of mass (m) and acceleration (a).

$$F = ma$$

(a) Velocity (v) is equal to the product of acceleration (a) and time (t).
(b) Weight (W) is equal to the product of mass (m) and gravity (g).
(c) The perimeter (P) of a rectangle is equal to the sum of twice the width (w) and twice the length (l).
(d) Energy (E) is equal to the product of the speed of light (c) squared and mass (m).
(e) The volume (V) of a cuboid is equal to the product of the length (l), width (w) and height (h).
(f) The diameter (D) of a circle is equal to the circumference (C) divided by pi (π).
(g) The gradient (m) is equal to the rise (y) divided by the run (x).
(h) The hypotenuse (h) is equal to the square root of the sum of the base (b) squared and the altitude (a) squared.

4. Use the following symbols in the questions below.

b = the weight of a banana
a = the weight of an apple
p = the weight of a plum

Examples Find the weight of the following quantities of fruit.

1. 6 bananas
$= 6b$

2. 3 bananas and 2 apples
$= 3b + 2a$

3. 5 apples and 1 plum
$= 5a + p$

Find the weight of the following quantities of fruit.

(a) 2 bananas and 6 apples
(b) 5 apples and 8 plums
(c) 7 plums and 10 bananas
(d) 1 apple and 1 plum
(e) 1 banana, 2 apples and 3 plums
(f) 6 plums, 9 apples and 15 bananas

5. Use the following symbols in the questions below.

c = the cost of a canary
b = the cost of a budgerigar

Examples Find the following amounts of money.

1. The cost of 2 canaries and 4 budgerigars.
$= 2c + 4b$

2. The change from \$50 if 3 canaries and 5 budgerigars are bought.
$= 50 - (3c + 5b)$

(a) Find the cost of 6 canaries and 7 budgerigars.
(b) Find the cost of 4 budgerigars and 6 canaries.
(c) Find the cost of a budgerigar and 2 canaries.
(d) Find the change from \$20 if 2 canaries are bought.
(e) Find the change from \$40 if 3 budgerigars are bought.
(f) Find the change from \$50 if 4 budgerigars and 2 canaries are bought.
(g) Find the change from \$100 if 4 canaries and 5 budgerigars are bought.
(h) Find the cost of buying 2 budgerigars, 5 canaries and a cage worth \$40.
(i) Find the cost of buying 4 budgerigars, 6 canaries and \$15 of feed.

6. Martin wanted to buy some CDs through an internet music company. The CDs will cost $40 each and there will be a $10 postal charge (for any number of CDs).

(a) Which of the following formulae could be used to calculate the ***total*** cost (C) of buying n CDs?

A $C = 10n + 40$ **B** $C = n + 40 + 10$
C $C = 40n + 10$ **D** $C = 40n$

(b) Find the cost of buying the following numbers of CDs.
(i) 1 CD (ii) 3 CDs (iii) 10 CDs

7. In a National Park there were 100 wombats. It was calculated that this population was increasing by 5 every year.

(a) Which of the following formulae could be used to calculate the total number (n) of wombats after x years?

A $n = 100x + 5$ **B** $n = 5x + 100$
C $n = 5 + x + 100$ **D** $n = 100 + x$

(b) Find the number of wombats after the following number of years.
(i) 2 years
(ii) 5 years
(iii) 20 years

(c) After how many years will there be 600 wombats?

8. A tank holds 200 litres of water. The water is be drained off at the rate of 2 litres every minute to water some plants.

(a) Which of the following formulae could be used to calculate the volume of water (V) remaining in the tank after it has been draining for t minutes?

A $V = 200 + 2t$ **B** $V = 200 - t$
C $V = 200 - (2 + t)$ **D** $V = 200 - 2t$

(b) Find how many litres of water are in the tank after it has been draining for the following number of minutes.
(i) 5 minutes (ii) 20 minutes (iii) 50 minutes

(c) After how many minutes will the tank be empty?

9. Lara wanted to buy some tickets for a concert. The tickets cost \$30 each and there was a booking fee of \$10.

Examples

1. The cost of buying 3 tickets
$= 3 \times \$30 + \$10$
$=$ **\$100**

2. The cost of buying 8 tickets
$= 8 \times \$30 + \$10$
$=$ **\$250**

(a) Write a formula that could be used to find the total cost (C) of buying n tickets.
(b) Find the cost of buying the following number of tickets.
(i) 2 tickets (ii) 5 tickets (iii) 10 tickets
(c) How many tickets could be bought with \$220?

10. Caitlin has \$500 in the bank. She works weekends and puts \$50 in the bank each week.
(a) Write a formula that could be used to calculate the amount of money (A) Caitlin has in the bank after w weeks.
(b) How much money will she have in the bank after the following number of weeks?
(i) 2 weeks (ii) 5 weeks (iii) 12 weeks
(c) How many weeks will it take Caitlin to have \$1000 in the bank?

11. Khaled bought 300 logs of wood to burn over winter. He finds that he burns 5 logs every day.
(a) Write a formula that could be used to calculate the number of logs of wood (N) he has remaining after d days of burning.
(b) Calculate how many logs he has remaining after burning wood for the following number of days.
(i) 5 days (ii) 10 days (iii) 15 days (iv) 20 days
(c) Copy and complete the following table showing the number of logs of wood remaining after burning for the days shown.

Number of days burning (d)	5	10	15	20	25	30	35	40
Number of logs remaining (N)								

(d) How many days will it take to burn all the wood?

12. Tomas can run at a speed of 2 metres every second.

(a) Write a formula that can be used to calculate the distance he can run (d) in t seconds.

(b) Calculate how far he can run in the following times.

(i) 10 seconds (ii) 30 seconds (iii) 1 minute (iv) 1 hour

(c) Copy and complete the following table showing how far Tomas can run in a given number of seconds.

Number of seconds (t)	10	20	30	40	50	60	70	80
Distance (metres) run (d)								

13. Use the following formula to answer the questions below.

$$A = B + 3$$

(a) Find A when B is:

(i) 0 (ii) 1 (iii) 8 (iv) 10 (v) 20

(b) Copy and complete the following table.

B	0	1	2	3	4	5	6	7	8	9	10
A											

14. For each of the formulae below, copy and complete the table shown in question 13.

(a) $A = 4B$ (b) $A = 2B + 1$ (c) $A = 3B + 4$

15. For each of the formulae below, copy and complete the following table.

N	2	4	6	8	10	12	14	16	18	20	22
M											

(a) $M = N - 2$ (b) $M = 2N - 3$ (c) $M = 4N - 8$

16. For each of the formulae below, copy and complete the following table.

x	1	4	7	12	15	30					
y							13	25	37	49	61

(a) $y = 2x + 3$ (b) $y = 3x - 5$ (c) $y = 4x + 5$

Addition and Subtraction

EXERCISE 8C

1. Simplify the following additions.

Examples

1. $a + a + a$
$= \mathbf{3a}$

2. $t + t + t + t + u + u$
$= \mathbf{4t + 2u}$

(a) $c + c + c + c + c$
(b) $x + x$
(c) $y + y + z + z + z + z$
(d) $m + m + m + m + m + n$
(e) $p + q + p + q + p + q$
(f) $f + f + f + g + f + f + f$
(g) $A + A + B + B + B + A$
(h) $a + b + c + a + b + c + a + b + c$

2. Simplify the following expressions.

Examples

1. $5a + 2a$
$= \mathbf{7a}$

2. $7t + t - 4t$
$= 8t - 4t$
$= \mathbf{4t}$

(a) $2x + 3x$
(b) $4c + 5c$
(c) $5p + p$
(d) $2q + 3q + 4q$
(e) $v + 6v + 2v$
(f) $9w - 3w$
(g) $8a - 6a$
(h) $7g - 6g$
(i) $10j - 3j - 2j$
(j) $3p + 7p - 5p$
(k) $4s - 3s + 5s$
(l) $6r - 4r - 2r$
(m) $6y - y + 7y$
(n) $7d - 7d + 2d$
(o) $5e + 7e - 12e$

3. Simplify the following expressions.

Examples

1. $5a - 8a$
$= \mathbf{-3a}$

2. $4t - 7t + 9t$
$= -3t + 9t$
$= \mathbf{6t}$

3. $-3k - 5k$
$= \mathbf{-8k}$

(a) $6x - 9x$
(b) $5c - 9c$
(c) $p - 3p$
(d) $-2q - 3q$
(e) $-4v + 5v + 2v$
(f) $6w - 10w + 2w$
(g) $-a - 5a$
(h) $3g - 7g - 6g$
(i) $5j - 2j - 6j$
(j) $-3p + 3p - 5p$
(k) $2s + 3s - 8s$
(l) $-6r - 4r - 2r$

4. Match the ***like terms*** in the following list of terms.

Like terms have the same ***pronumerals (letters)***

Example

$2a \quad 3b \quad 6a^2 \quad b \quad -\frac{1}{2}a \quad -ab \quad 12a^2 \quad 12ab \quad 6a \quad 2b$

The groups of ***like terms*** are:

$2a, -\frac{1}{2}a, 6a$	$3b, b, 2b$	$6a^2, 12a^2$	$-ab, 12ab$

$2m \quad 4n^2 \quad 6mn \quad n \quad -4m \quad 6m^2 \quad \frac{1}{4}n^2 \quad 4n \quad \frac{1}{4}mn \quad 4m^2n \quad 3m^2 \quad -3m^2n$

5. Simplify the following expressions by collecting the ***like terms***.

Examples **1.** $5x + 3y + 8x - y$

$= 5x + 3y + 8x - y$

$= \mathbf{13x + 2y}$

2. $4a + 7 + 1 - 3a$

$= 4a + 7 + 1 - 3a$

$= \mathbf{a + 8}$

(a) $2a + 3b + 5a + 4b$
(b) $6m + 5n - 2n - 3m$
(c) $8p + 5q - 7p - 5q$
(d) $6t + 7u + t - 6u$
(e) $7 + 5a + 2 + 3a$
(f) $6c + 7d - d + c$
(g) $2g + 5m + m - 2g$
(h) $5v + 7y + 2y - 3y$
(i) $3n^2 + 2n + 2n^2 - n$
(j) $2p + 3r - 3r - 2p$

6. Simplify the following expressions by collecting the ***like terms***.

Examples **1.** $5a + 3b - a - 5b$

$= 5a + 3b - a - 5b$

$= \mathbf{4a - 2b}$

2. $5x - 7 - 1 - 8x$

$= 5x - 7 - 1 - 8x$

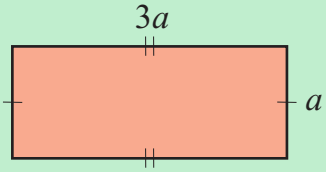
$= \mathbf{-3x - 8}$

(a) $5x + 2y - 3x - 5y$
(b) $3a + 4b - 5a - 2b$
(c) $3p - 2q - p - 3q$
(d) $2d - 6f - 4d + 8f$
(e) $-3m + 2 - m - 4$
(f) $-a - b - a - b$
(g) $-3p + 4 - 5p + 6$
(h) $2q - 4q + 6q - 8q$
(i) $-2y - 2y - 2y - 2y - 2y$
(j) $-2k + 3m + 6k - 9m$

7. Find the perimeters of the following shapes.

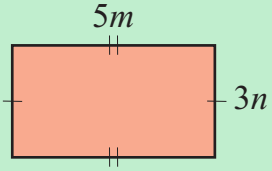
Examples

1.



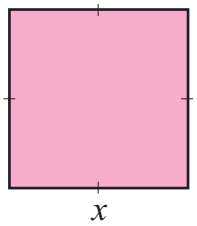
Perimeter = all the side lengths added
$= 3a + a + 3a + a$
$= \mathbf{8a}$

2.

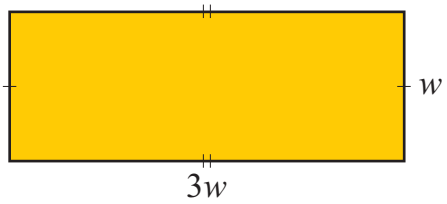


Perimeter = all the side lengths added
$= 5m + 3n + 5m + 3n$
$= \mathbf{10m + 6n}$

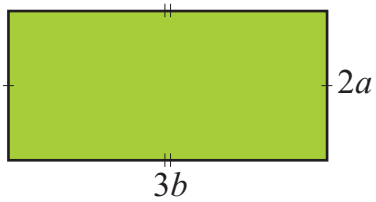
(a)



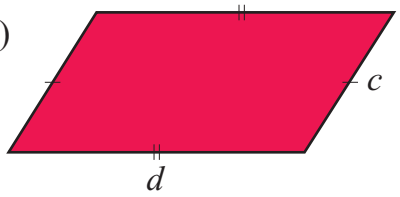
(b)



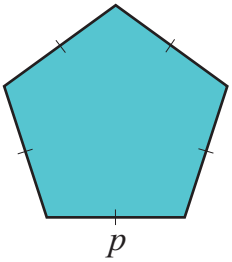
(c)



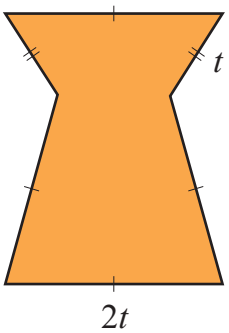
(d)



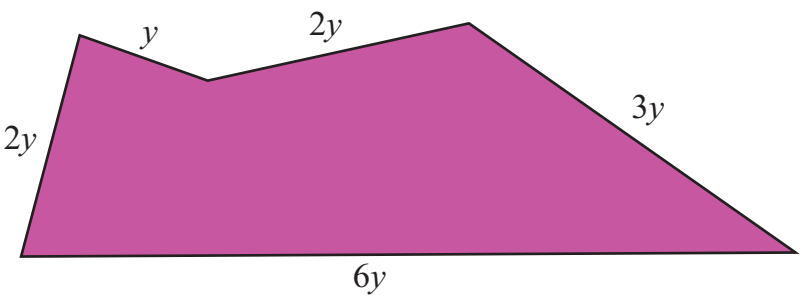
(e)



(f)



(g)



8. (a) If a banana weighs b, find the weight of the following bunches of bananas:
(i) 4 bananas (ii) 8 bananas (iii) 12 bananas
(b) If these three bunches of bananas are put into a bag, what is the total weight of the bag of bananas?

9. Sinead saves x dollars in the first month after starting a job. She then saves $2x$ dollars in the next month and $3x$ in the following month.
(a) What is the total amount Sinead saved in the three months?
(b) If Sinead had $8x$ dollars in the bank before starting the job, how much has she in her bank after the three months?
(c) Sinead then withdraws $5x$ dollars from her bank account. How much does she now have in her bank account?

10. Duncan, Tina, Emily and Peter went fishing and all caught one fish.
Duncan's fish weighed x kg.
Tina's fish weighed twice the weight of Duncan's fish.
Emily's fish weighed 2 kg more than Duncan's fish.
Peter's fish weighed 1 kg less than Duncan's fish.
(a) Find an expression for the weight of:
(i) Tina's fish (ii) Emily's fish (iii) Peter's fish
(b) Find an expression for the total weight of the four fish.
(c) If the total weight of the four fish is 16 kg, find the weight of Duncan's fish.

11. Hamish, Imogen, Caz and Dean were abseiling down a rock face.
Hamish took t minutes to abseil down the rocks.
Imogen took 10 minutes longer than Hamish.
Caz took twice as long as Hamish.
Dean took 10 minutes less than Caz.
(a) Find an expression for the time taken to abseil down the rock face by:
(i) Imogen (ii) Caz (iii) Dean
(b) Find an expression for the total time taken by the four rock climbers to abseil down the rock face.
(c) If it took a total of 2 hours for the four rock climbers to abseil down the rock face, how long did Hamish take?

Multiplication and Division

EXERCISE 8D

1. Simplify the following expressions.

Examples **1.** $2 \times 6 \times m$ $= \mathbf{12}\boldsymbol{m}$ **2.** $5 \times a \times 3 \times b$ $= \mathbf{15}\boldsymbol{ab}$ **3.** $4x \times 7y$ $= \mathbf{28}\boldsymbol{xy}$

(a) $3 \times 4 \times a$ (b) $m \times n$ (c) $4 \times v \times 5 \times w$

(d) $3 \times 7 \times g \times h$ (e) $8 \times d \times c \times 6$ (f) $10 \times m \times 3 \times n \times p$

(g) $6a \times 3b$ (h) $2x \times 11y$ (i) $4a \times 3b \times 2c$

2. Simplify the following expressions.

Examples **1.** $\dfrac{20m}{5}$ $= \dfrac{{}^{4}\cancel{20}m}{{}_{1}\cancel{5}}$ $= \mathbf{4}\boldsymbol{m}$ **2.** $18k \div 2$ $= \dfrac{18k}{2}$ $= \dfrac{{}^{9}\cancel{18}k}{{}_{1}\cancel{2}}$ $= \mathbf{9}\boldsymbol{k}$

(a) $\dfrac{16p}{8}$ (b) $\dfrac{24s}{3}$ (c) $\dfrac{30t}{6}$ (d) $\dfrac{48ab}{6}$

(e) $28m \div 7$ (f) $15n \div 5$ (g) $27p \div 9$ (h) $32xyz \div 8$

3. There are x metres of material on a roll.
(a) How many metres of material would be on 12 rolls?
(b) The material from the 12 rolls is divided into 6 equal portions. How long is each of these portions of material?

4. Callista takes t minutes to run a lap of the local park.
(a) How long would it take Callista to run 6 laps of the park?
(b) Callista runs 6 laps of the park every night for a week. How many minutes does she run in the week?

5. A school hall has rows of seats set up for a concert.
(a) If there are x seats in each row and 20 rows in the hall, how many seats are in the hall?
(b) How much money does the school make if the hall is booked out and each ticket cost \$4?

Indices

EXERCISE 8E

1. Simplify the following terms.

Examples

1. $a \times a \times a \times a$
$= \boldsymbol{a^4}$

2. $3 \times m \times m \times m \times 5 \times n \times n$
$= \boldsymbol{15m^3n^2}$

3. $5a \times 2a$
$= \boldsymbol{10a^2}$

(a) $b \times b \times b$
(b) $2 \times t \times t \times t \times t \times t \times t \times t \times t$
(c) $4 \times p \times 3 \times p$
(d) $8 \times q \times q \times q \times 2 \times q \times q$
(e) $2w \times 2w \times 2w \times 2w \times 2w$
(f) $3 \times c \times c \times c \times c \times 2 \times c \times c \times 5$
(g) $3 \times a \times b \times 2 \times a \times b \times 2 \times a$
(h) $d \times 4d \times e \times 3e \times 2d \times 2d \times de$

2. Simplify the following terms.

Examples

1. $a^3 \times a^2$
$= \overbrace{a \times a \times a} \times \overbrace{a \times a}$
$= \boldsymbol{a^5}$

2. $2m^2 \times 6m^5$
$= 2 \times \overbrace{m \times m} \times 6 \times \overbrace{m \times m \times m \times m \times m}$
$= \boldsymbol{12m^7}$

A quicker way is to ***add*** the indices.

1. $a^{\textcircled{3}} \times a^{\textcircled{2}}$ ($3 + 2 = 5$)
$= \boldsymbol{a^5}$

2. $2m^{\textcircled{2}} \times 6m^{\textcircled{5}}$ ($2 + 5 = 7$)
$= 12m^7$

3. $p^7 \times p^3$
$= \boldsymbol{p^{10}}$

4. $4n^2 \times 6n^9$
$= \boldsymbol{24n^{11}}$

5. $3k^6 \times 2k$
$= \boldsymbol{6k^7}$

(a) $m^6 \times m^2$
(b) $q^5 \times q^5$
(c) $b^3 \times b^2 \times b^4$
(d) $3g^4 \times 2g^7$
(e) $h^4 \times h$
(f) $2k^3 \times 2k^3 \times 3k^3$
(g) $2n^3 \times 6n^5$
(h) $3c^7 \times c$
(i) $2p^7 \times 3p \times p^4$
(j) $4w^3 \times 2w^6 \times w$
(k) $5y^6 \times y^2 \times y^3 \times y$
(l) $a^x \times a^y$

3. Simplify the following terms.

Examples

$(x^3)^4$
$= x^3 \times x^3 \times x^3 \times x^3$
$= \boldsymbol{x^{12}}$

A quicker way is to ***multiply*** the indices.

$3 \times 4 = 12$

1. $(x^{③④})$
$= \boldsymbol{x^{12}}$

2. $(a^2b^3c)^5$
$= \boldsymbol{a^{10}b^{15}c^5}$

(a) $(g^4)^2$ (b) $(m^3)^3$ (c) $(v^7)^2$ (d) $(p^3)^5$
(e) $(q^2)^3 \times (q^3)^4$ (f) $(a^2b^3)^4$ (g) $(fg^4)^5$ (h) $(c^2e^4)^6$
(i) $(a^3)^4 \times (a^2)^5$ (j) $(ab^2c^3)^4$ (k) $(n^{10})^{10}$ (l) $(3a^4)^2$

4. Simplify the following terms.

Examples ***Anything to the power of 0 equals 1***

1. w^0
$= \mathbf{1}$

2. $(ab)^0$
$= \mathbf{1}$

3. $(6u)^0$
$= \mathbf{1}$

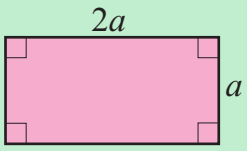
4. $7m^0$
$= 7 \times 1$
$= \mathbf{7}$

(a) g^0 (b) $(m^3)^0$ (c) $(u^2v^7)^0$ (d) $h^0 + k^0$
(e) $(24t)^0$ (f) $5a^0$ (g) $(2h)^0 + 2h^0$ (h) $5p^0 - 2k^0$
(i) $3k^0 \times 2m^0$ (j) $(4e)^0 \times r^0$ (k) $5^0 - 3^0$ (l) $5x^3y^0$

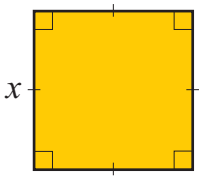
5. Write expressions for the ***shaded area*** in each of the following shapes.

Example

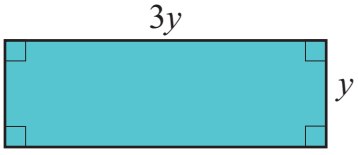
Area = length × width
$= 2a \times a$
$= \boldsymbol{2a^2}$



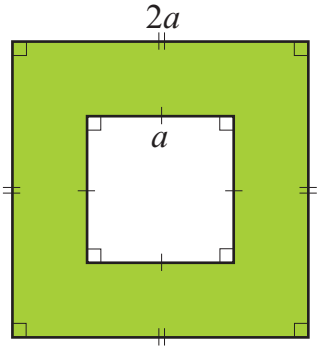
(a)



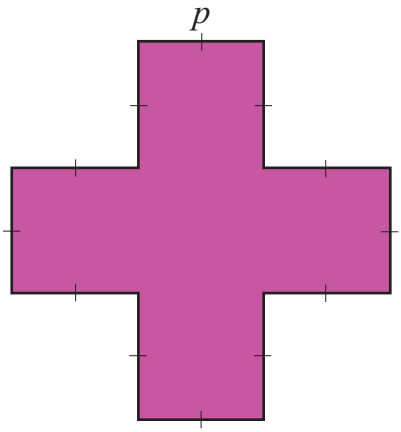
(b)



(c)



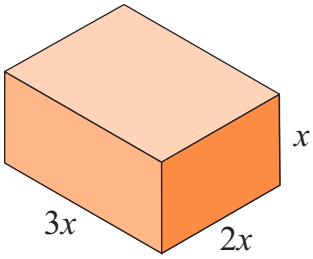
(d)



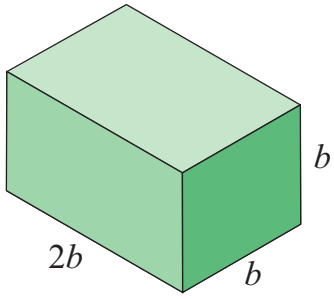
6. Write an expression for the volume of each of the following objects.

Volume of a rectangular prism = length × width × height

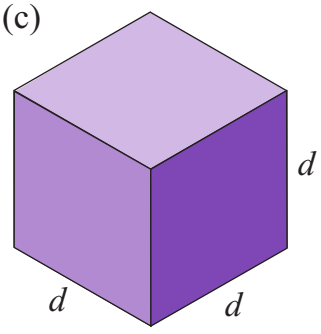
(a)



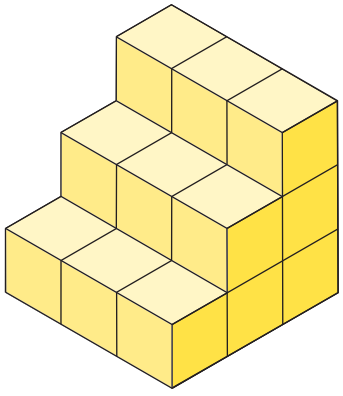
(b)



(c)



(d)

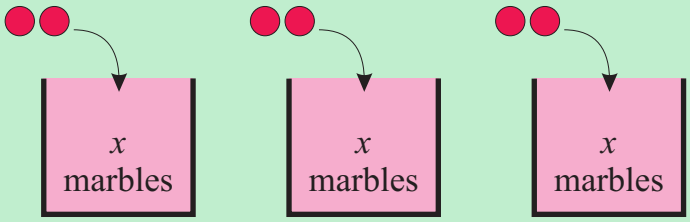


Each small cube in this object has side length m.

Expanding Algebraic Expressions

Example

Three boxes each have x marbles in them.
If 2 marbles are added to each box, what is the total number of marbles?
Write answer ***two ways***.



First Way

Step 1 Before the two marbles are added to each box there is a total of $\mathbf{3}x$ marbles in the boxes.

Step 2 If 2 marbles are added to each box, then **6** marbles have been added.

Step 3 There is a total of $\mathbf{3}x + \mathbf{6}$ marbles.

Second Way

Step 1 After the 2 marbles are added to each box there are $(x + \mathbf{2})$ marbles in each box.

Step 2 There are 3 boxes so the total number of marbles is $\mathbf{3}(x + \mathbf{2})$.

The two answers are: $\mathbf{3}x + \mathbf{6}$ and $\mathbf{3}(x + \mathbf{2})$

EXERCISE 8F

Write the answers to the following problems ***two ways***.

1. Jayne is organising a birthday party for her four year old son.
She has 8 bags with y sweets in each bag.
If she adds another 3 sweets to each bag, what is the total number of sweets?

2. A tennis coach has 5 bags of tennis balls. There are b balls in each bag and she adds another 4 balls to each bag.
What is the total number of tennis balls?

3. Kellie has 6 containers of raspberries that each weigh r grams.
If she adds 20 grams of raspberries to each container, what will be the total weight of the raspberries?

4. Thommo has 3 bags of Easter eggs each with e eggs in them.
If he eats 4 from each bag, how many Easter eggs will he have left?

5. A pet shop has 7 puppies that each cost p dollars. If the price is ***reduced*** by $5 on each puppy, how much money will the pet shop make if it sells the 7 puppies?

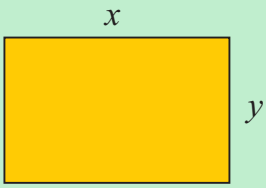
6. A golfing expert has 12 bags of golf balls that he uses for practice. Each bag contains g golf balls. If 5 balls are ***lost*** from each bag, how many golf balls has he got left?

Example

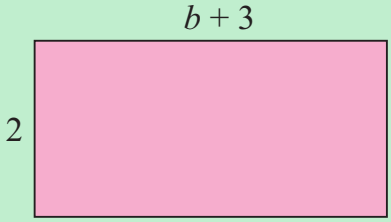
The area of a rectangle is equal to its length multiplied by its width.

Area = length × width

Area = 6 × 3
= **18 m²**

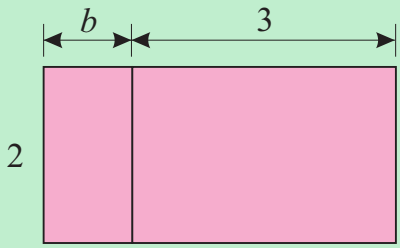


Area = $x \times y$
= $\boldsymbol{xy}$

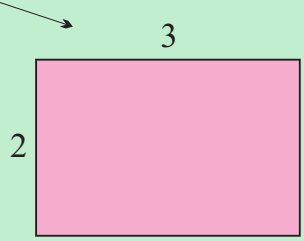


Area = $2 \times (b + 3)$
= $\boldsymbol{2(b + 3)}$

This rectangle can be divided into two rectangles:



Area = $2 \times b$
= $\boldsymbol{2b}$



Area = 2×3
= **6**

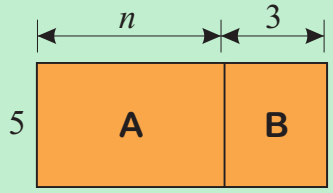
The area of the total rectangle equals the sum of the area of the two smaller rectangles:

$$\boldsymbol{2(b + 3) = 2b + 6}$$

EXERCISE 8G

1. For the following rectangles:
(i) write the area of each of the smaller rectangles
(ii) write the total area of the larger rectangle

Example



(i) Area of smaller rectangles:

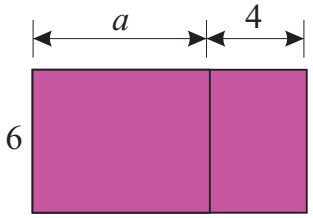
A $5 \times n = \mathbf{5n}$

B $5 \times 3 = \mathbf{15}$

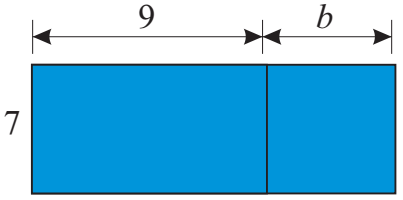
(ii) Area of larger rectangle:

$5 \times (n + 3) = \mathbf{5(n + 3)}$

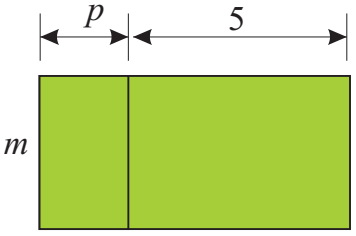
(a)



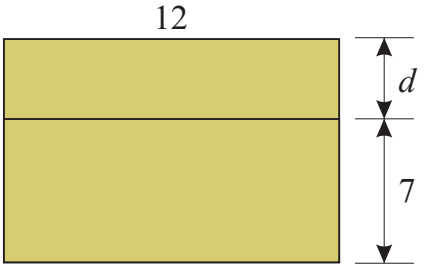
(b)



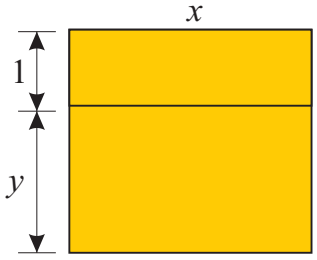
(c)



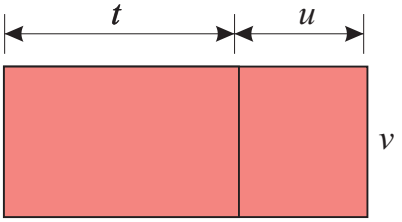
(d)



(e)

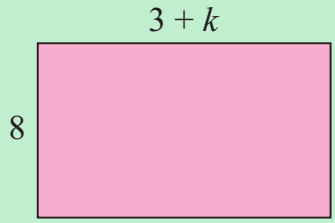


(f)



2. Write the area of the following rectangles two ways.

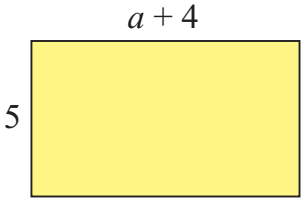
Example



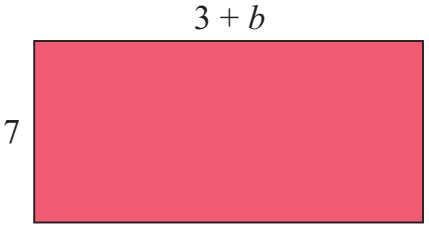
First way: Area = $8 \times (3 + k)$
$= \mathbf{8(3 + k)}$

Second way: Area = $8 \times 3 + 8 \times k$
$= \mathbf{24 + 8k}$

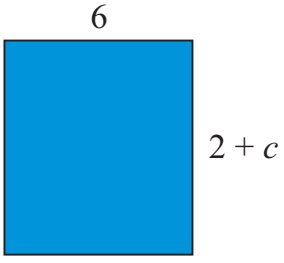
(a)



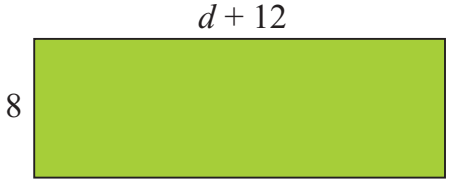
(b)



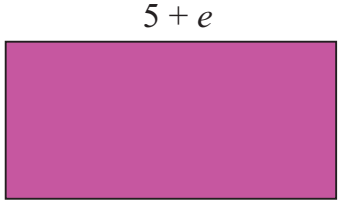
(c)



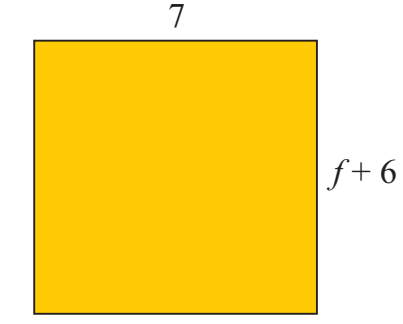
(d)



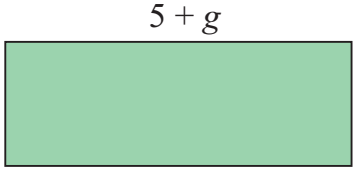
(e)



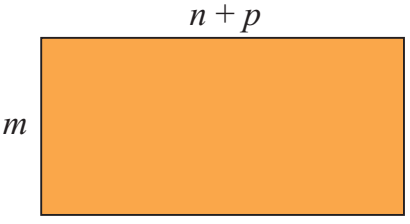
(f)



(g)



(h)

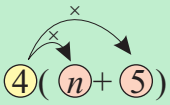


Example

Expand the following algebraic expression.

$$4(n + 5)$$

A simple way of doing this is to multiply each term in the bracket by the term in front of the bracket.



$= 4 \times n + 4 \times 5$

$= \mathbf{4n + 20}$

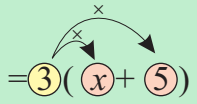
This is called the ***distributive law***.

EXERCISE 8H

1. Expand the following algebraic expressions.

Examples

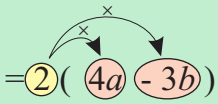
1. $3(x + 5)$



$= 3 \times x + 3 \times 5$

$= \mathbf{3x + 15}$

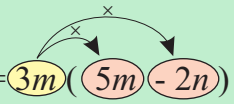
2. $2(4a - 3b)$



$= 2 \times 4a + 2 \times \mathbf{-}3b$

$= \mathbf{8a - 6b}$

3. $3m(5m - 2n)$



$= 3m \times 5m + 3m \times \mathbf{-}2n$

$= \mathbf{15m^2 - 6mn}$

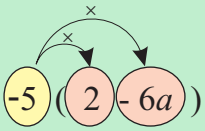
(a) $4(a + 2)$
(b) $3(x - 4)$
(c) $2(m + p)$
(d) $6(3c + 1)$
(e) $2(e + 2f)$
(f) $5(3 - g)$
(g) $3(4 - 2t)$
(h) $4(3a - 2b)$
(i) $x(y + 2)$
(j) $a(a + 2)$
(k) $2m(m + 1)$
(l) $3v(2v - 3w)$
(m) $5t(4t + 5u)$
(n) $3p(p + 4q)$
(o) $2a(3a + 6b)$
(p) $3a(b - c)$
(q) $6x(5y - 3z)$
(r) $7k(6m - 5n)$
(s) $7a(2a + 3b)$
(t) $5t(4t - 7u)$
(u) $2p(4p - 5q)$
(v) $8y(3x + 2y)$
(w) $10g(g - 3h)$
(x) $9x(2xy - 3yz)$

2. Expand the following algebraic expressions.

Remember: if two negative numbers are multiplied the answer is a positive number.

Examples

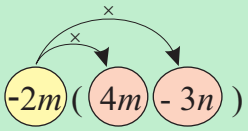
1. $-5(2 - 6a)$



$= -5 \times 2 + -5 \times -6a$

$= \mathbf{-10 + 30a}$

2. $-2m(4m - 3n)$



$= -2m \times 4m + -2m \times -3n$

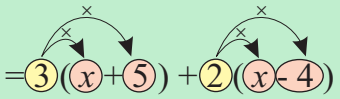
$= \mathbf{-8m^2 + 6mn}$

(a) $-4(a + 2)$ (b) $-3(x + 7)$ (c) $-3(2y + 9)$
(d) $-3(m - n)$ (e) $-3(r - 8)$ (f) $-k(m - n)$
(g) $-2(7a + 3b)$ (h) $-7(4t - 5u)$ (i) $-2p(5p - 2q)$
(j) $-8y(3x + 4y)$ (k) $-10g(2g - 3h)$ (l) $-9x(2xy - 4z)$

3. Expand and simplify the following expressions.

Examples

1. $3(x + 5) + 2(x - 4)$



$= 3x + 15 + 2x - 8$

$= \mathbf{5x + 7}$

2. $4(a + 6) - 2(a - 5)$

$= 4a + 24 - 2a + 10$

$= \mathbf{2a + 34}$

(a) $3(2x + 3) + 2(3x + 1)$ (b) $4(3m + 4) + 2(2m - 5)$
(c) $4(3t + 2) + 2(2t - 3)$ (d) $3(2g - 3) + 2(3g + 4)$
(e) $5(2t - 1) + 3(3t - 4)$ (f) $2(3z - 2) + 6(4z - 2)$
(g) $2(2w + 2) - 3(w + 2)$ (h) $3(2p + 4) - 2(2p + 3)$
(i) $4(3h - 3) - 2(2h - 3)$ (j) $5(3q + 2) - 4(3q - 5)$

Factorisation

EXERCISE 8I

1. Write ***all*** the factors of the following numbers.

> ***Example*** 18
> All the factors of 18 are **1**, **2**, **3**, **6**, **9, 18**

(a) 12 (b) 36 (c) 27 (d) 80 (e) 112
(f) 56 (g) 300 (h) 23 (i) 144 (j) 171

2. Find the factors that are common to the following pairs of numbers.

> ***Example*** 20, 30
> The factors of 20 are 1, 2, 4, 5, 10
> The factors of 30 are 1, 2, 3, 5, 6, 10, 15
> The ***common factors*** are **1**, **2**, **5**, **10**

(a) 12, 16 (b) 15, 27 (c) 16, 20 (d) 24, 36 (e) 56, 60

3. Write the highest common factor (H.C.F.) for each of the pairs of numbers in question 2.

> ***Example*** The highest common factor of 20 and 30 is **10**

4. Find the highest common factor of the following pairs of numbers.

(a) 4, 12 (b) 30, 6 (c) 8, 12 (d) 18, 27 (e) 21, 14
(f) 24, 32 (g) 35, 45 (h) 48, 36 (i) 36, 72 (j) 84, 49

5. Find the highest common factor of the following terms.

Examples

1. $3a, ab$

$3a = 3 \times a$ $\quad ab = a \times b$

highest common factor = ***a***

2. $2x^2, 3xy$

$2x^2 = 2 \times x \times x$ $\quad 3xy = 3 \times x \times y$

highest common factor = ***x***

(a) $4m, mn$ (b) $6b, ab$ (c) $pq, 3p$ (d) $8u, 3uv$
(e) $2ab, 3ac$ (f) $4xy, 5yz$ (g) $6mp, 7mq$ (h) $2de, 5ef$
(i) a^2, ac (j) $m^2, 8m$ (k) $5p, 2p^2$ (l) $xy, 2y^2$
(m) $9h^2, 5h$ (n) $2a^2, 3abc$ (o) $m^2n, 5n$ (p) $3x^2y, 5yz$

Find the highest common factor of the following terms.

Examples

Find the highest common factor of the ***numbers*** in each term and using this write the factors.

1. $3ab, 12bc$

Highest common factor of 3 and 12 is **3**.

$3ab = 3 \times a \times b$ $\quad 12bc = 3 \times 4 \times b \times c$

highest common factor $= 3 \times b$
$= \mathbf{3b}$

2. $10mn, 8m^2$

Highest common factor of 10 and 8 is **2**.

$10mn = 2 \times 5 \times m \times n$ $\quad 8m^2 = 2 \times 4 \times m \times m$

highest common factor $= 2 \times m$
$= \mathbf{2m}$

6. Find the highest common factor of the following terms.

(a) $4xy$, $8x$ (b) $5ab$, $15ac$ (c) $12pq$, $6qr$
(d) $7cd$, $28de$ (e) $36yz$, $4z$ (f) $45ab$, $20bd$
(g) $18mn$, $27m$ (h) $32x^2$, $24x$ (i) $54ac$, $24cd$
(j) $18xy$, $36y^2$ (k) $12mn$, $36m^2$ (l) $28hg$, $35hi$
(m) $26fg$, $14g^2$ (n) $72st$, $54tu$ (o) $56kl$, $88l^2$

7. Find the highest common factor of the following terms.

(a) $10y$, 8 (b) $36c$, $24a$ (c) $5p$, $6pq$
(d) $12d$, $9de$ (e) $48yz$, $20xz$ (f) $30ab$, $49bd$
(g) $16mn$, $20m^2$ (h) $18x$, $72y$ (i) $14ac$, $24cd$
(j) $15xy$, $36y^2$ (k) $18n$, $36m^2$ (l) $24hi$, $35hg$
(m) $28fg$, $16g^2$ (n) $27tu$, $63vw$ (o) $65kl$, $85l$

Factorisation is the ***opposite*** of ***expansion*** of algebraic expressions.

To factorise an algebraic expression the ***highest common factor*** of all the terms must be found first.

Example 1

Factorise the following algebraic expression.

$$3x + 6$$

Step 1 Find the highest common factor of each term.

In this example the highest common factor of $3x$ and 6 is **3**.

Step 2 Write each term as a product of two factors with one being the highest common factor.

$$3x + 6$$
$$= 3 \times x + 3 \times 2$$

Step 3 Take the highest common factor out of each term. Write the highest common factor outside a bracket with the remaining factors inside the bracket.

$$3 \times x + 3 \times 2$$
$$= \mathbf{3(x + 2)}$$

Example 2

Factorise the following algebraic expression.

$8a + 12b$

Highest common factor (H.C.F.) of $8a$ and $12b$ is **4**

$= 4 \times 2a + 4 \times 3b$

$= \mathbf{4(2a + 3b)}$

Example 3

Factorise the following algebraic expression.

$24mn - 18n$

$= 6n \times 4m - 6n \times 3$ (H.C.F. of $24mn$ and $18n$ is $\mathbf{6n}$)

$= \mathbf{6n(4m - 3)}$

Example 4

Factorise the following algebraic expression.

$4x + 20x^2$

$= 4x \times 1 + 4x \times 5x$ (H.C.F. of $4x$ and $20x^2$ is $\mathbf{4x}$)

$= \mathbf{4x(1 + 5x)}$

Example 5

Factorise the following algebraic expression.

$18d - 25de$

$= d \times 18 - d \times 25e$ (H.C.F. of $18d$ and $25de$ is $\boldsymbol{d}$)

$= \boldsymbol{d(18 - 25e)}$

Example 6

Factorise the following algebraic expression.

$32pq + 16qr$

$= 16q \times 2p + 16q \times r$ (H.C.F. of $32pq$ and $16qr$ is $\mathbf{16q}$)

$= \mathbf{16q(2p + r)}$

8. Factorise the following algebraic expressions.

(a) $6m + 8p$ (b) $5x - 2xy$ (c) $9 - 18a$
(d) $3ab + a$ (e) $10d - 15e$ (f) $6ab + 12a$
(g) $x^2 + 7x$ (h) $8u - 20u^2$ (i) $14bc - 21de$
(j) $32ab + 24b$ (k) $m - 8mn$ (l) $9c^2 - 6cd$
(m) $24pq + 36qr$ (n) $12r^2 - 6r$ (o) $21kl + 49lr$
(p) $36xy - 30yz$ (q) $20gh + 9gi$ (r) $45ab - 72bd$

Example 7

Factorise the following algebraic expression.

$8m^2n + 16mn - 32mn^2p$

The H.C.F. of these three terms is $\mathbf{8mn}$.

$= 8mn \times m + 8mn \times 2 - 8mn \times 4np$

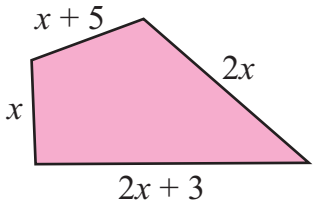
$= \mathbf{8mn(m + 2 - 4np)}$

9. Factorise the following algebraic expressions.

(a) $3x^2y - 6xy + 12xy^2$ (b) $16m - 12mp + 24m^2$
(c) $15abc + 24b^2c - 9bcd$ (d) $8ghi + 6hij + 12fhi$
(e) $36pqr^2 - 24qrs + 30prt$ (f) $72m^2np - 45mnp + 63mn^2p$
(g) $42xyz^2 + 35yz^2 - 14x^2yz$ (h) $32cd^2e + 24c^2de - 8cde^2$
(i) $18ab^2c - 9abc + 45abc^2$ (j) $10x^2y^2z^2 + 12xy^2z - 4x^2yz^2$

PROBLEM SOLVING

1. The perimeter of this shape is 80 metres. Find x.

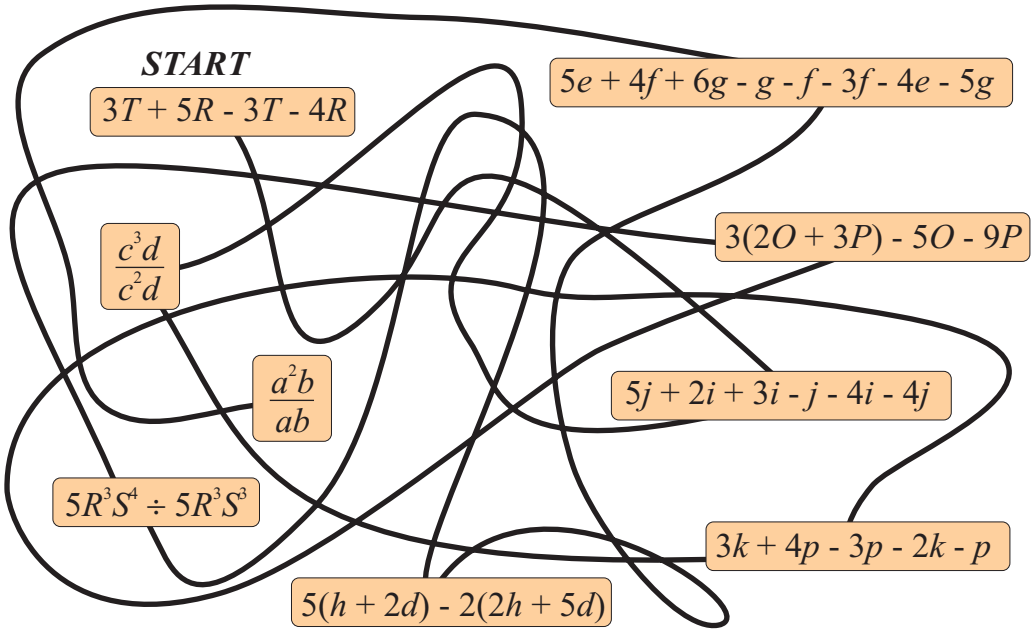


2. Hans had \$3905 in the bank. He wanted to build his bank account to \$8000 to buy a car, so he decided to save some money each month. In the first month he saved x dollars. Each month after that, he put into the bank ***double*** the amount he banked in the previous month. What amount does he need to save in the first month (x) so he has \$8000 in his bank account after a ***year***?

PUZZLE

Follow the path below (beginning at the ***START***) simplifying the expressions in the order that you find them on the path.
The solutions will spell the answer to this riddle.

Who was the Irishman who bounced off the wall?



CHAPTER REVIEW

1. Simplify the following expressions.

(a) $x + x + x + x$ (b) $7p + 8p$ (c) $3y - 3y$

(d) $4a + a$ (e) $5m + 6n - 4m - 2n$ (f) $-3a + 5b - 2a - 3b$

2. An apple costs d dollars.

(a) Write an expression for the cost of a bag of 8 apples.

(b) Write an expression for the cost of 4 of these bags of apples.

3. Sam is y years old. Sam's brother, Dion, is 3 years older than Sam. Sam's sister, Anthea, is 2 years younger than Sam. Sam's mother, Nea, is three times Sam's age and his father, Ernie, is 1 year younger than Sam's mother.

(a) Write an expression for the age of each of the members of Sam's family.

(b) Write an expression for the sum of all their ages.

4. Simplify the following expressions.

(a) $2 \times a \times 3$ (b) $3 \times n \times 4 \times m$ (c) $n \times n \times n \times n \times n$

(d) $3x \times 4y \times 2x \times 6x$ (e) $m^5 \times m^4$ (f) $3j^2 \times 5j^3$

(g) $(e^4)^2$ (h) $(u^2v^3)^5$ (i) $(c^2)^3 \times (c^4)^2$

(j) b^0 (k) $(2h)^0$ (l) $3g^0 + 5g^0$

5. Expand the following expressions.

(a) $5(x + 3)$ (b) $4(n - 2)$ (c) $3(2a + 5b)$

(d) $4m(2m + 3n)$ (e) $-3(2 + 5v)$ (f) $-3x(2x - 5y)$

6. Expand and simplify: $2(x + 7) + 3(4x - 3)$

7. Factorise the following algebraic expressions.

(a) $8m + 4p$ (b) $7a - 3ab$ (c) $12 - 24x$

(d) $5xy + y$ (e) $12e - 18f$ (f) $8ab + 24a$

(g) $2x + x^2$ (h) $9u - 15u^2$ (i) $18mn - 24mp$

(j) $36ab + 24b$ (k) $m - 7mn$ (l) $10c^2 - 8cd$

(m) $24pq + 48qr$ (n) $18r^2 - 8r$ (o) $27kl + 54lr$