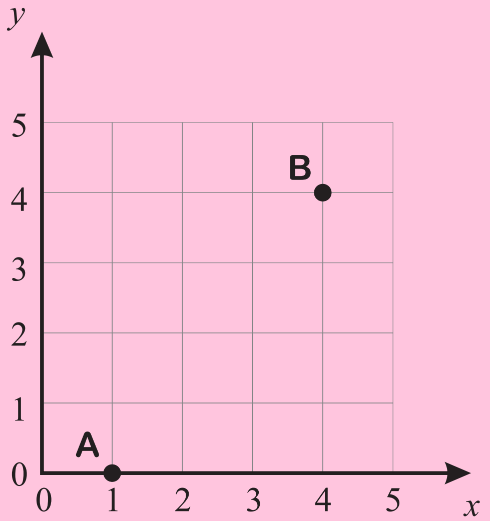
# Straight Lines

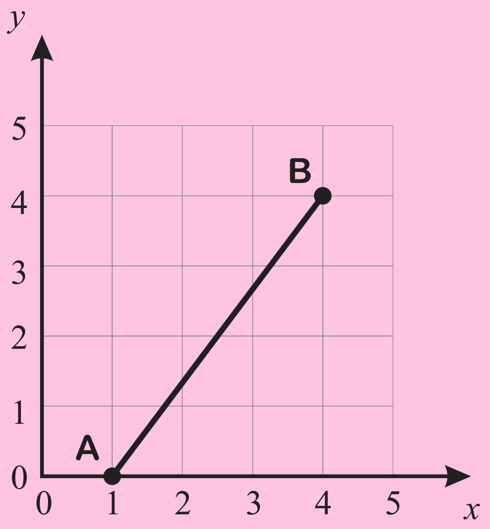
## Distance Between Two Points on a Straight Line

***Example 1***

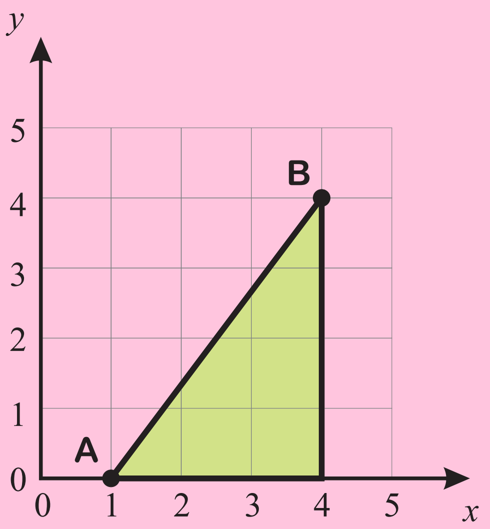
Find the distance between points **A** and **B** shown on the graph below.



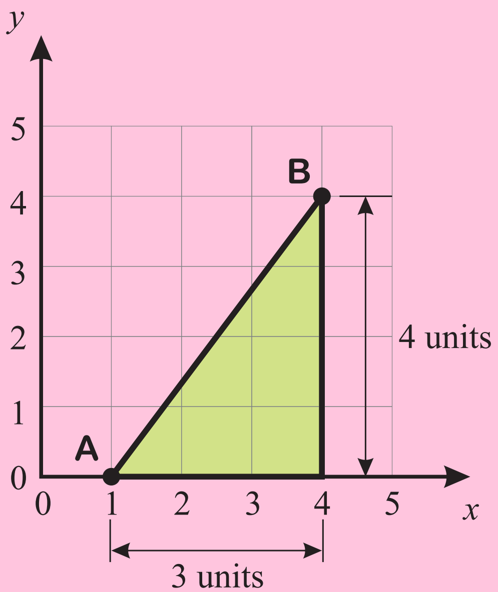

***Step 1*** Draw a line between **A** and **B**.



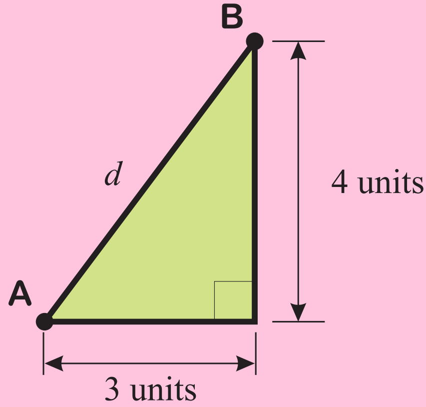
***Step 2*** Draw a right-angled triangle using this line as the long side of the triangle - this side is called the hypotenuse.




***Step 3*** Find the side lengths of the two shorter sides.



***Step 4*** Label the longest side *d*.

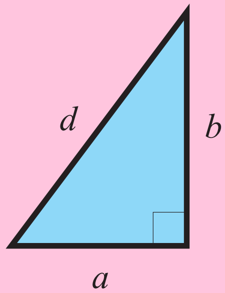



***Step 5*** Pythagoras' Theorem can be used to find this length.

Pythagoras' Theorem states:

$$\boldsymbol{d^2 = a^2 + b^2}$$

$$\boldsymbol{d = \sqrt{a^2 + b^2}}$$




Substituting in the values for this triangle.

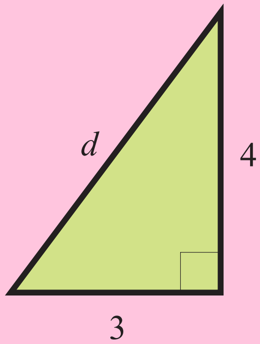
$$d = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

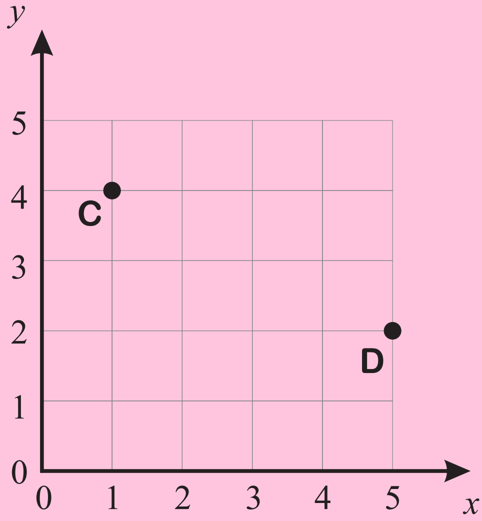
$$= \sqrt{25}$$

$$\boldsymbol{d = 5}$$

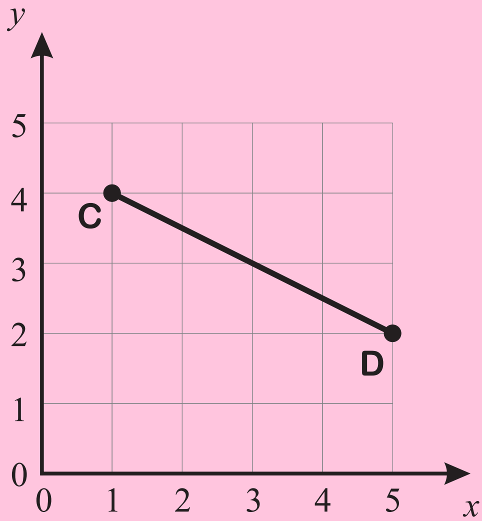


***Example 2***

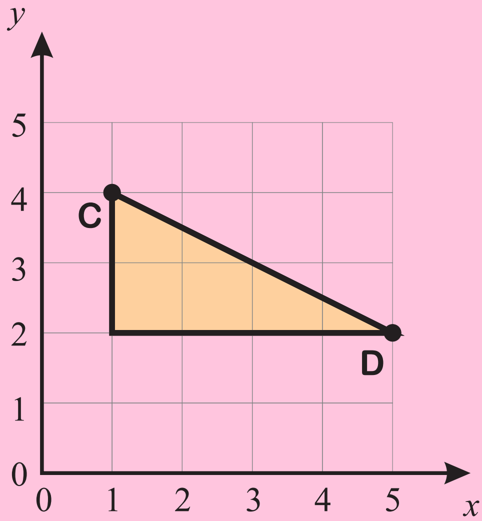
Find the distance between points **C** and **D** shown on the graph below.



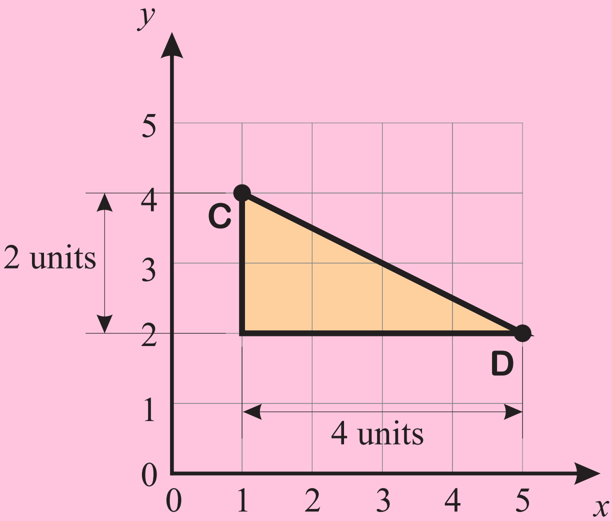

***Step 1***



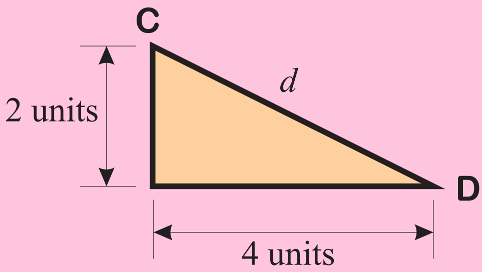
***Step 2***



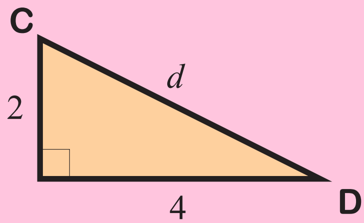

***Step 3***



***Step 4***




***Step 5***




$$d = \sqrt{a^2 + b^2}$$

$$= \sqrt{4^2 + 2^2}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

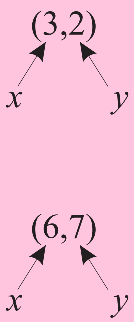
$$\boldsymbol{d = 4.4721}$$

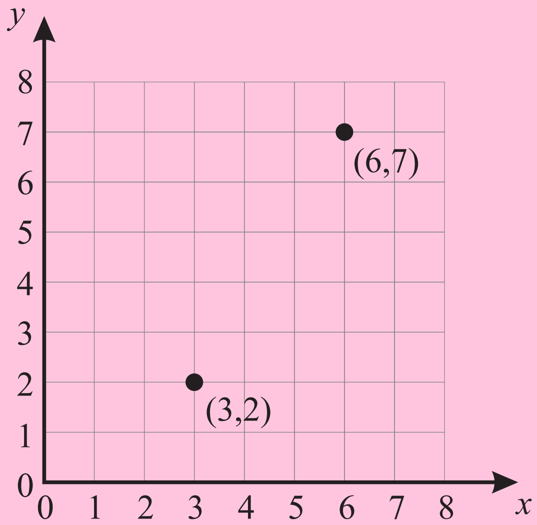
$\boldsymbol{d = 4.5}$ **(one decimal place)**

***Example 3***

Find the distance between the points with coordinates (3,2) and (6,7).

***Step 1*** Locate these points on a Cartesian plane. Remember the first number in a set of coordinates is the $x$-ordinate and the second is the $y$-ordinate.
Make sure the graph is large enough to include both points.

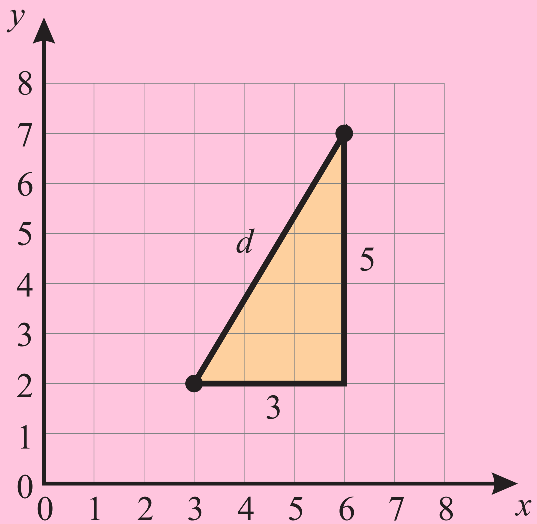



***Step 2*** Follow the steps 1-5 from the previous examples.

$$d = \sqrt{a^2 + b^2}$$
$$= \sqrt{3^2 + 5^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$
$$d = 5.8309$$

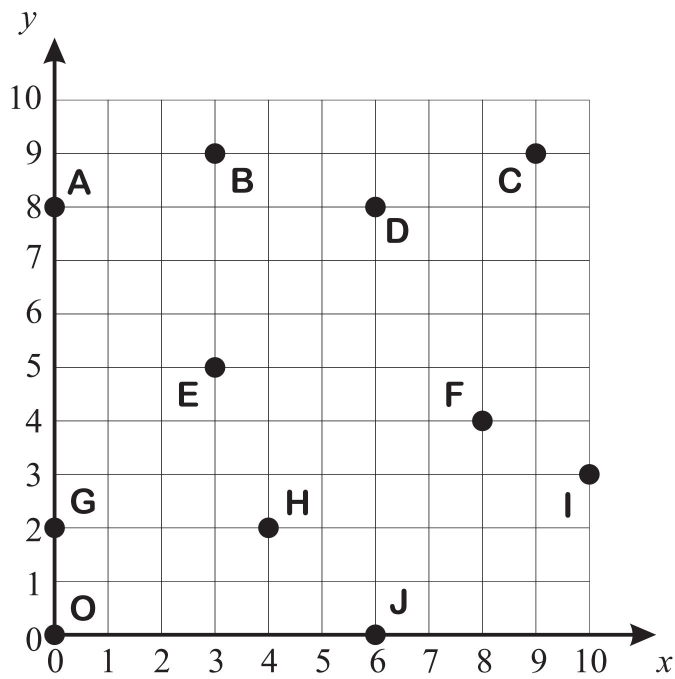
$\boldsymbol{d}$ **= 5.8 (one dec. pl.)**



## EXERCISE 8A

**1.** Find the distance between the following pairs of points shown on this Cartesian plane. Give answers correct to one decimal place.

(a) **G** and **E**
(b) **H** and **I**
(c) **A** and **J**
(d) **H** and **J**
(e) **B** and **F**
(f) **O** and **B**
(g) **C** and **E**
(h) **C** and **H**

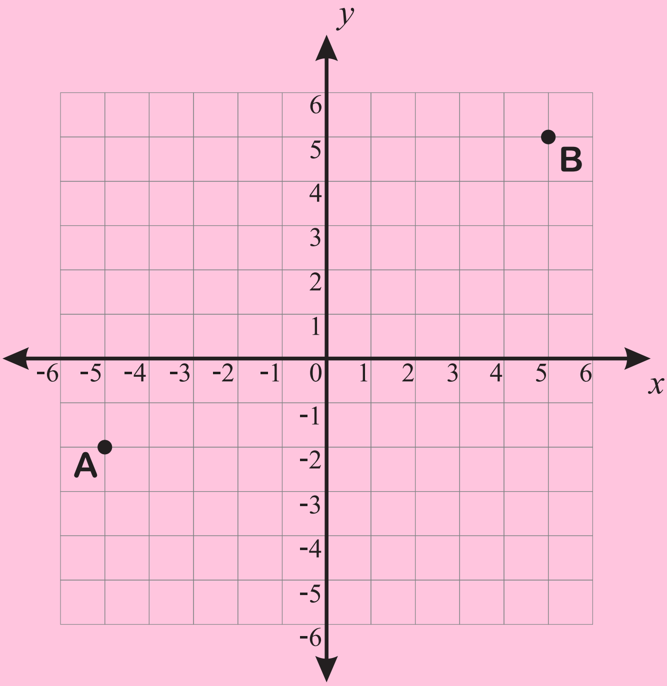



**2.** (a) Draw a Cartesian plane with $x$ and $y$ axes of 0 to10.
(b) Locate the following points on the Cartesian plane.

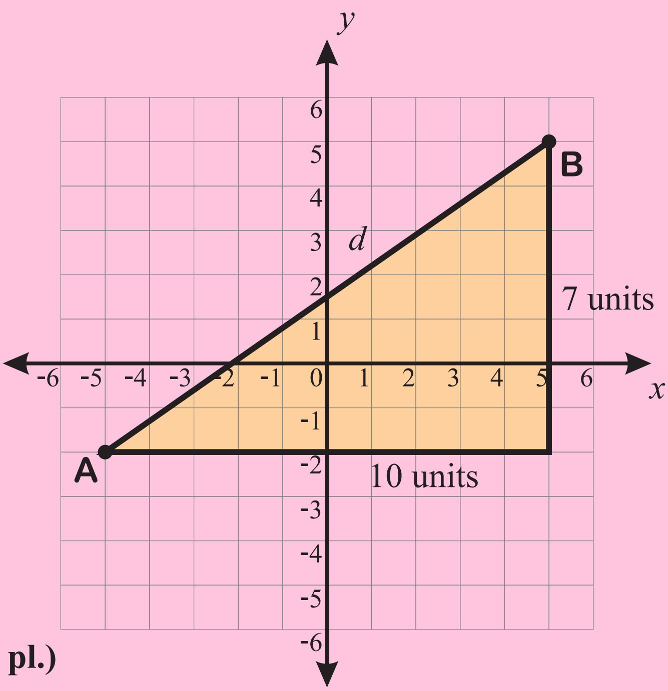
**K** = (1,9), **L** = (0,3), **M** = (9,2), **N** = (8,7)
**P** = (3,2), **Q** = (5,0), **R** = (5,6), **S** = (1,8)

(c) Find the distance between the following pairs of points. Give answers correct to one decimal place.
(i) **K** and **P**
(ii) **L** and **M**
(iii) **Q** and **N**
(iv) **R** and **M**
(v) **M** and **S**

***Example*** Find the distance between the points **A** and **B** shown on the Cartesian plane below.




Follow the steps from the previous examples.




$$d = \sqrt{a^2 + b^2}$$

$$= \sqrt{10^2 + 7^2}$$

$$= \sqrt{100 + 49}$$

$$= \sqrt{149}$$

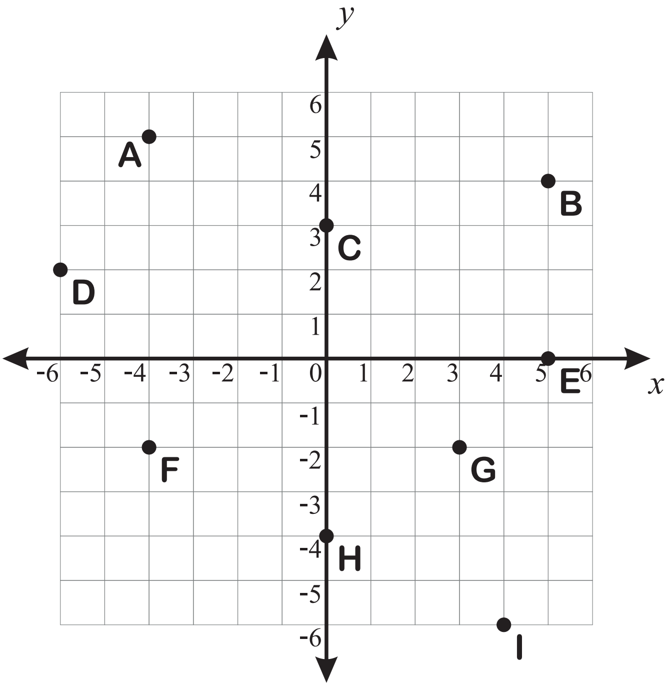
$$d = 12.2065$$

$\boldsymbol{d}$ **= 12.2 (one dec. pl.)**

## EXERCISE 8B

**1.** Find the distance between the following pairs of points shown on this Cartesian plane.

(a) **D** and **B**
(b) **F** and **C**
(c) **A** and **E**
(d) **D** and **G**
(e) **F** and **B**
(f) **A** and **I**
(g) **H** and **B**



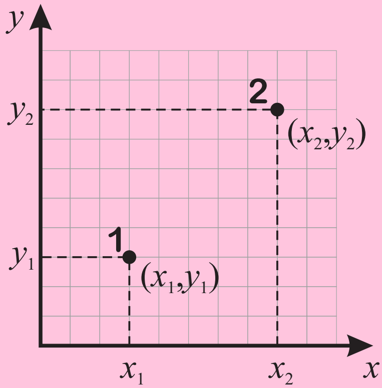

**2.** (a) Draw a Cartesian plane with $x$ and $y$ axes of -8 to 8.
(b) Locate the following points on the Cartesian plane.

**K** = (-1,7), **L** = (0,-3), **M** = (-5,-2), **N** = (7,6)
**P** = (5,-4), **Q** = (4,0), **R** = (-7,6), **S** = (-1,-8)

(c) Find the distance between the following pairs of points.
Give answers correct to one decimal place.
(i) **K** and **Q**
(ii) **N** and **M**
(iii) **Q** and **S**
(iv) **L** and **P**
(v) **R** and **P**

# Formula to Find the Distance Between Two Points

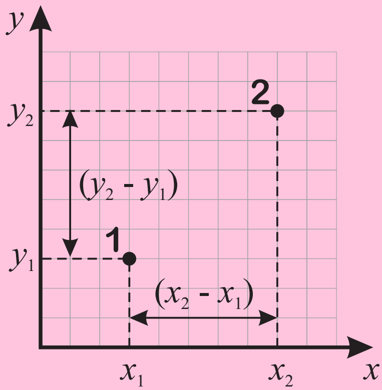
Consider two points **1** and **2**.
Let the coordinates of point **1** = $(x_1,y_1)$.
Let the coordinates of point **2** = $(x_2,y_2)$.




It can be seen that:

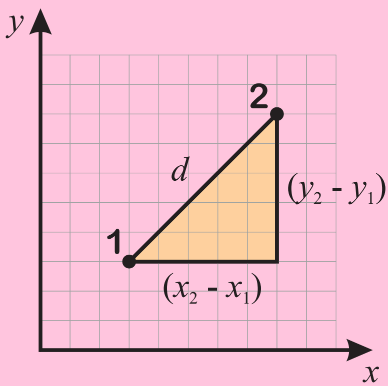
the horizontal distance between the two points = $(x_2 - x_1)$.

the vertical distance between the two points = $(y_2 - y_1)$.




The triangle can then be drawn and Pythagoras' Theorem used to find the formula to find $d$.

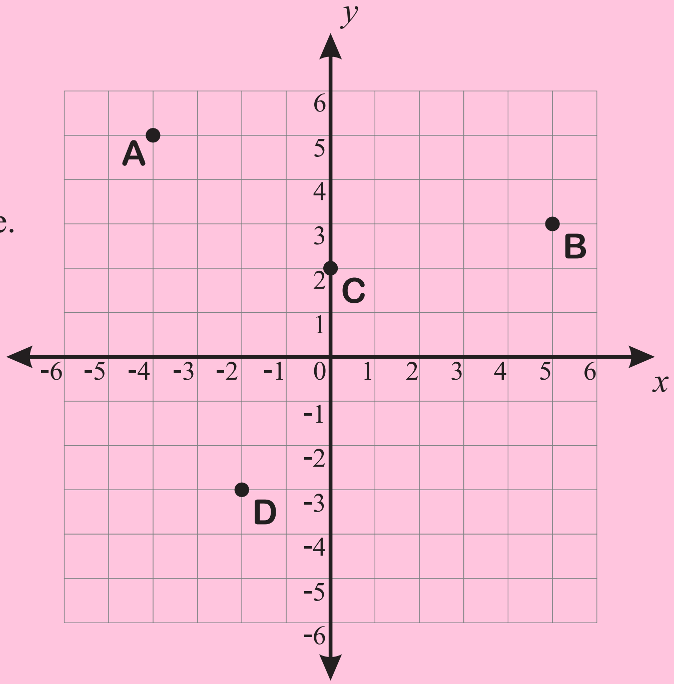
$$d = \sqrt{a^2 + b^2}$$




$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

***Example 1***

Find the distance between the following pairs of points shown on this Cartesian plane. Give answers correct to one decimal place.




(a) **A** and **B**
(b) **B** and **C**
(c) **A** and **D**

***Answers***

(a) Let point **A** = point **1**. Coordinates = $(x_1,y_1)$ = (-4,5)

Let point **B** = point **2**. Coordinates = $(x_2,y_2)$ = (5,3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5 - -4)^2 + (3 - 5)^2}$$

$$= \sqrt{(5 + 4)^2 + (-2)^2}$$

$$= \sqrt{9^2 + (-2)^2}$$

$$= \sqrt{81 + 4}$$

$$= \sqrt{85}$$

$$= 9.219$$

= **9.2** (one decimal place)

Either point can be used as point **1**. The answer will be the same.

(b) Let point **B** = point **1**. Coordinates = $(x_1,y_1) = (5,3)$

Let point **C** = point **2**. Coordinates = $(x_2,y_2) = (0,2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0 - 5)^2 + (2 - 3)^2}$$

$$= \sqrt{(-5)^2 + (-1)^2}$$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$= 5.099$$

$= \mathbf{5.1}$ (one decimal place)

(c) Let point **A** = point **1**. Coordinates = $(x_1,y_1) = (-4,5)$

Let point **D** = point **2**. Coordinates = $(x_2,y_2) = (-2,-3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2 - -4)^2 + (-3 - 5)^2}$$

$$= \sqrt{(-2 + 4)^2 + (-8)^2}$$

$$= \sqrt{2^2 + (-8)^2}$$

$$= \sqrt{4 + 64}$$

$$= \sqrt{68}$$

$$= 8.246$$

$= \mathbf{8.2}$ (one decimal place)

***Example 2***

Find the distance between (-8,16) and (-11,7) on a Cartesian plane.

Let point **1** = (-8,16) = $(x_1,y_1)$

Let point **2** = (-11,7) = $(x_2,y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-11 - -8)^2 + (7 - 16)^2}$$

$$= \sqrt{(-11 + 8)^2 + (-9)^2}$$

$$= \sqrt{(-3)^2 + (-9)^2}$$

$$= \sqrt{9 + 81}$$

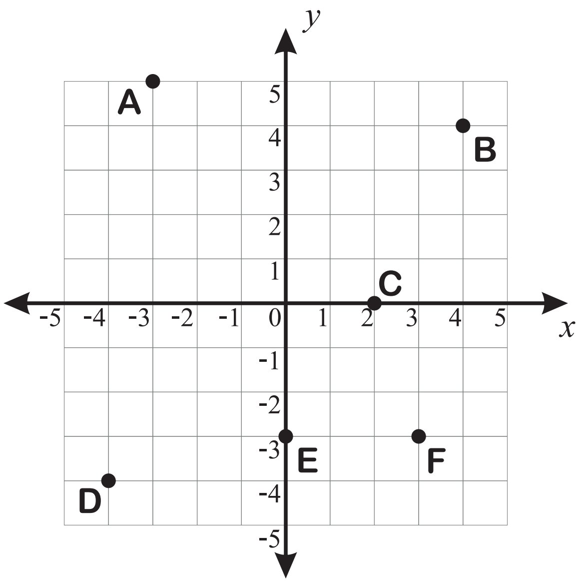
$$= \sqrt{90}$$

$$= 9.486$$

= **9.5** (one decimal place)

## EXERCISE 8C

**1.** Find the distance between the following pairs of points shown on this Cartesian plane.
Give answers correct to one decimal place.

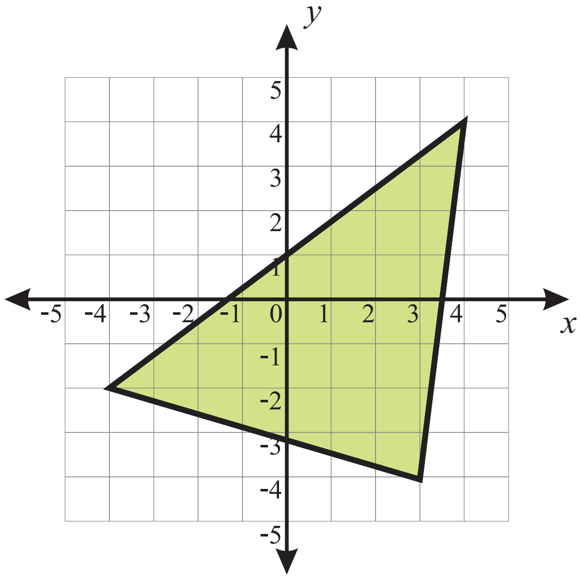



(a) **A** and **C**
(b) **B** and **D**
(c) **A** and **F**
(d) **E** and **C**
(e) **D** and **C**
(f) **B** and **F**

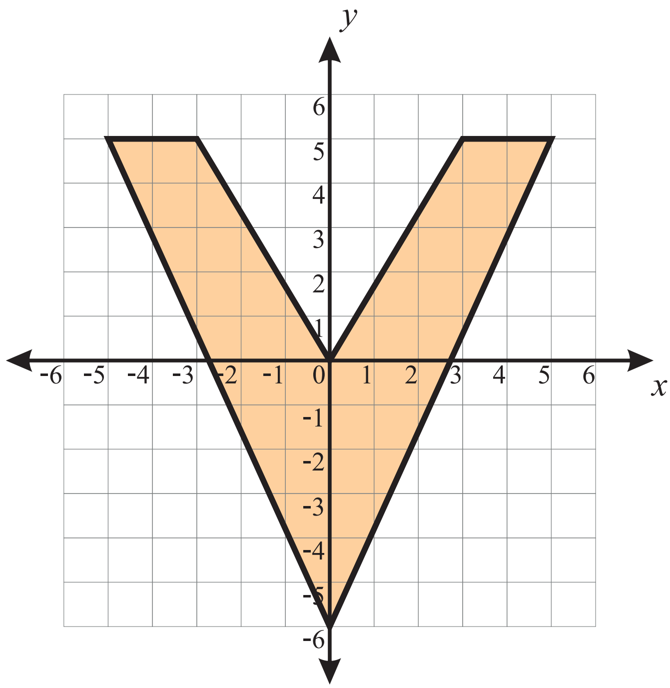
**2.** Find the distance between the following pairs of points.
Give answers correct to one decimal place.

(a) (6,13) and (3,10)
(b) (-4,15) and (7,-14)
(c) (-9,0) and (9,-18)
(d) (-7,-19) and (16,13)
(e) (17,-11) and (-23,14)
(f) (-29,12) and (-17,-15)

**3.** Find the perimeter of the triangle shown on this Cartesian plane.
Give answer correct to one decimal place.




**4.** Find the perimeter of the shape shown on this Cartesian plane.
Give answer correct to one decimal place.

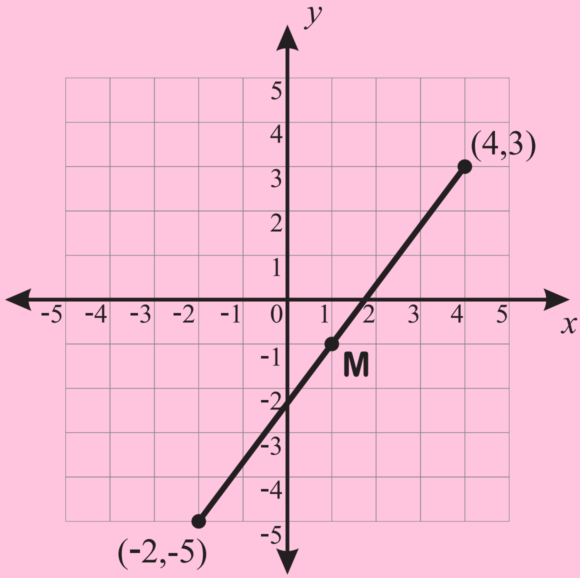


# Midpoint of the Line Connecting Two Points (Line Segment)

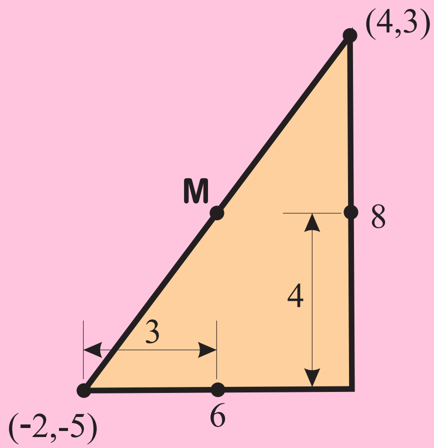
***Example 1***

Find the coordinates of the midpoint of the line connecting points (-2,-5) and (4,3).

The coordinates of the midpoint (**M**) of the line connecting two points on a Cartesian plane (line segment) can be found by following the steps below.




***Step 1*** Draw a line connecting the two points and form a right-angled triangle. Find the side lengths of the triangle. Calculate half of each side length.



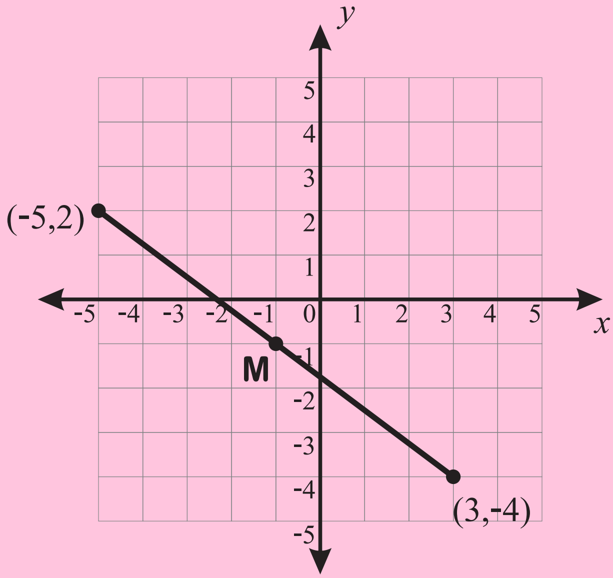

***Step 2*** Use these half lengths to find the coordinates the midpoint **M**.

Coordinates of **M** = (-2 + 3,-5 + 4)

= **(1,-1)**

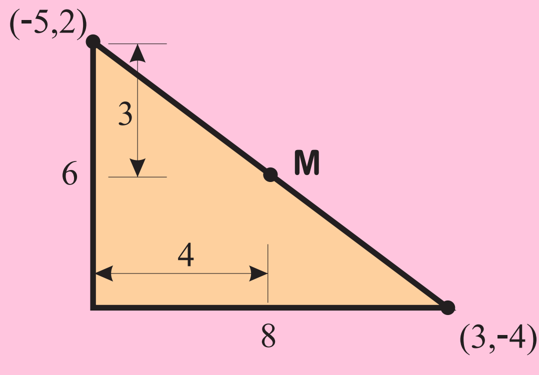
***Example 2***

Find the coordinates of the midpoint of the line connecting points (-5,2) and (3,-4).




In this example it can be seen that the coordinates of the midpoint will be:

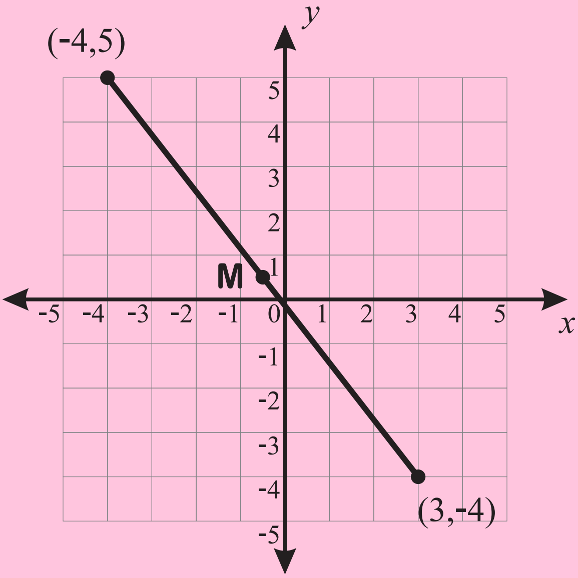
(-5 + 4, 2 - 3)

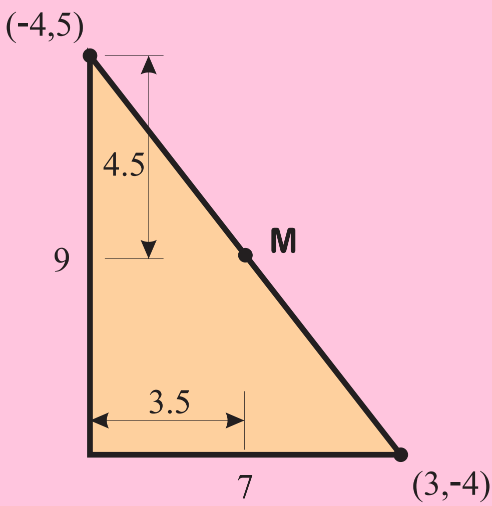



Coordinates of **M** = **(-1,-1)**

***Example 3***

Find the coordinates of the midpoint of the line connecting points (-4,5) and (3,-4).




In this example it can be seen that the coordinates of the midpoint will be:

(-4 + 3.5, 5 - 4.5)

Coordinates of **M** = **(-0.5,0.5)**

## EXERCISE 8D

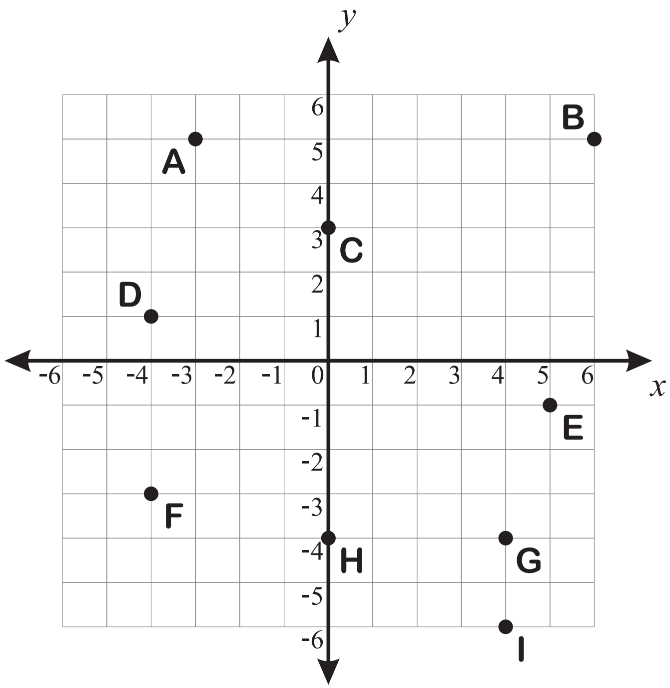
**1.** (a) Draw a Cartesian plane with $x$ and $y$ axes of -8 to 8.

(b) Find the coordinates of the midpoint of the line segment connecting each of the following pairs of points.

(i) (2,4) and (4,8)
(ii) (0,1) and (6,7)
(iii) (-3,2) and (7,4)
(iv) (0,7) and (6,1)
(v) (-5,5) and (3,-3)
(vi) (-1,-7) and (7,7)
(vii) (2,0) and (7,6)
(viii) (0,8) and (7,3)
(ix) (-4,1) and (3,7)
(x) (-6,-1) and (-1,-7)
(xi) (-5,6) and (6,-7)

**2.** Find the midpoint of the line segment connecting each of the following pairs of points shown on this Cartesian plane.

(a) D and B
(b) F and C
(c) A and E
(d) D and G
(e) F and B
(f) A and I
(g) H and B



# Formula to Find the Coordinates of the Midpoint of a Line Segment

The coordinates of the midpoint of the line segment connecting the points with coordinates $(x_1,y_1)$ and $(x_2,y_2)$ are:

$$\left(\frac{x_1 + x_2}{2} \;,\; \frac{y_1 + y_2}{2}\right)$$

***Example***

Find the coordinates of the midpoint of the line segment connecting the point (-5,-9) and (13,8).

$(x_1,y_1) = (-5,-9)$ $\qquad$ $(x_2,y_2) = (13,8)$

$$\left(\frac{x_1 + x_2}{2} \;,\; \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 13}{2} \;,\; \frac{-9 + 8}{2}\right)$$

$$= \left(\frac{8}{2} \;,\; -\frac{1}{2}\right)$$

$$= \mathbf{(4,-0.5)}$$

## EXERCISE 8E

Find the coordinates of the midpoint of the line segment that connects the following pairs of points.

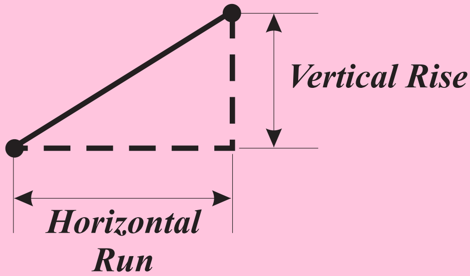
**1.** (2,8) and (10,12)
**2.** (0,5) and (14,6)
**3.** (-3,7) and (-5,6)
**4.** (-9,7) and (9,15)
**5.** (11,-7) and (-4,4)
**6.** (8,19) and (24,25)
**7.** (-13,-7) and (25,-16)
**8.** (-9,7) and (-25,36)
**9.** (18,-9) and (-18,9)
**10.** (12,-17) and (-24,44)
**11.** (13.6,7.9) and (-8.2,15.8)
**12.** (-15.7,-21.1) and (8.4,32.8)

# Gradient

***Definition***

The ***gradient*** of the line connecting two points is defined as:

$$\frac{\text{the vertical } \textbf{\textit{rise}} \text{ between the two points}}{\text{the horizontal } \textbf{\textit{run}} \text{ between the two points}}$$

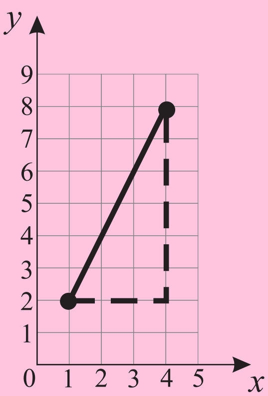


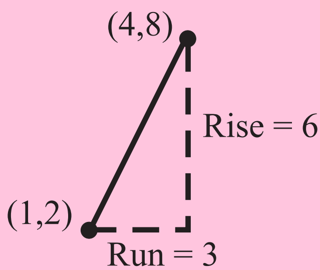

The symbol for gradient is $m$

$$\textbf{Gradient} = m = \frac{\textbf{Rise}}{\textbf{Run}}$$

***Example***

Find the gradient of the line connecting the points (1,2) and (4,8).




$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

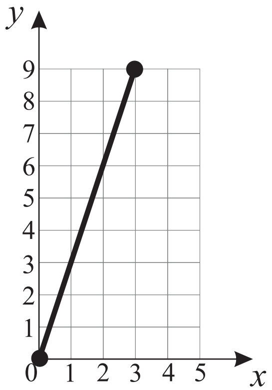
$$m = \frac{6}{3}$$

$$\boldsymbol{m = 2}$$

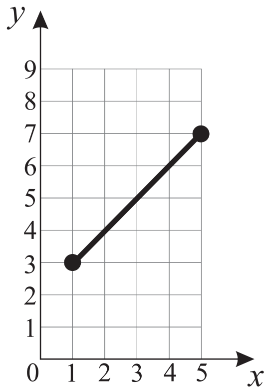
## EXERCISE 8F

**1.** Find the gradient of the lines shown on the graphs below.

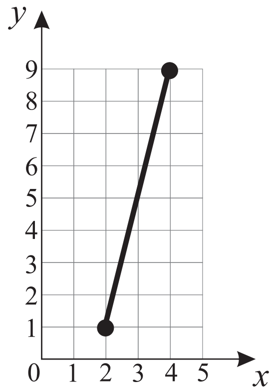
(a)




(b)




(c)

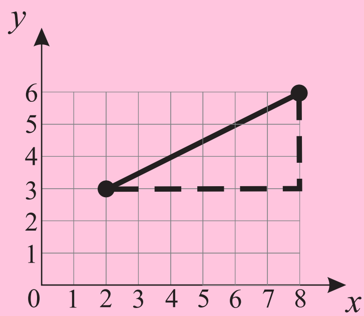



**2.** Find the gradient of the lines connecting the following pairs of points.

(a) (0,0) and (3,6)
(b) (1,0) and (6,5)
(c) (2,3) and (5,6)
(d) (3,2) and (6,8)
(e) (0,4) and (2,12)
(f) (3,1) and (9,7)
(g) (4,2) and (6,8)
(h) (3,0) and (11,16)

**3.** Find the gradient of the lines connecting the following pairs of points.

***Example*** (2,3) and (8,6)




Rise = 3
Run = 6

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

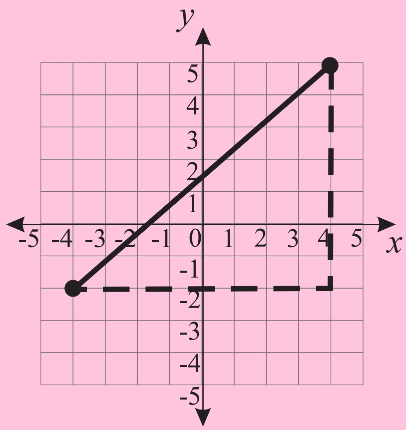
$$m = \frac{3}{6}$$

$$\boldsymbol{m = \frac{1}{2}}$$

(a) (0,0) and (4,2)
(b) (1,2) and (7,5)
(c) (2,3) and (8,6)
(d) (1,4) and (13,8)
(e) (0,4) and (10,7)
(f) (2,5) and (11,11)

**4.** Find the gradient of the lines connecting the following pairs of points.

***Example*** (-4,-2) and (4, 5)




Rise = 7
Run = 8

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

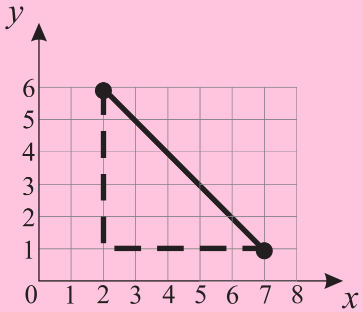
$$\boldsymbol{m = \frac{7}{8}}$$

(a) (-1,2) and (4,7)
(b) (-4,-2) and (1,8)
(c) (3,-2) and (10,5)
(d) (-1,2) and (7,6)
(e) (-2,-3) and (5,1)
(f) (-5,3) and (7,7)
(g) (-4,-4) and (6,8)
(h) (-1,-6) and (4,-1)
(i) (-3,0) and (0,5)
(j) (-8,5) and (-1,8)

***Negative Gradient***

If a line slopes down when going from left to right it has a ***negative*** gradient. It has a ***negative rise***.

***Example*** Find the gradient of the line joining (2,6) and (7,1)



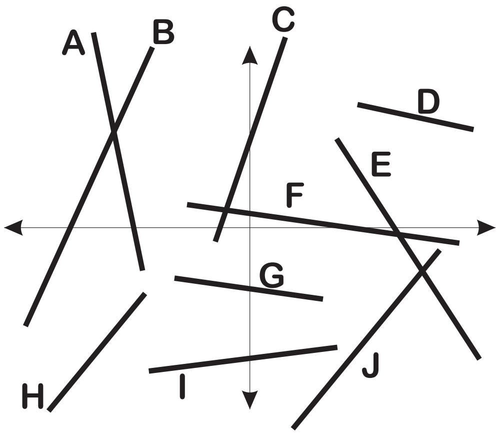

Rise = -5
Run = 5

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{-5}{5}$$

$$\boldsymbol{m = -1}$$

**5.** Which of the lines on this graph have a ***negative*** gradient.




**6.** Plot the following pairs of points on a graph and connect each pair with a straight line. State which of these lines will have a negative gradient and calculate these negative gradients.

(a) (1,4) and (5,0)
(b) (2,8) and (4,4)
(c) (-2,1) and (3,-4)
(d) (-1,-1) and (3,1)
(e) (-2,-3) and (2,-5)
(f) (0,3) and (5,0)
(g) (2,0) and (6,-2)
(h) (-3,-4) and (4,0)

## Calculating Gradient

The gradient of the line connecting points $(x_1,y_1)$ and $(x_2,y_2)$ can be found by using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

***Examples***

Find the gradient of the line connecting the following pairs of points.

**1.** (2,-3) and (5,7)

$(x_1,y_1) = (2,-3)$
$(x_2,y_2) = (5,7)$

$$m = \frac{7 - (-3)}{5 - 2}$$
$$= \frac{7 + 3}{3}$$
$$\boldsymbol{m = \frac{10}{3}}$$

**2.** (-5,4) and (-2,-3)

$(x_1,y_1) = (-5,4)$
$(x_2,y_2) = (-2,-3)$

$$m = \frac{(-3) - 4}{(-2) - (-5)}$$
$$= \frac{-3 - 4}{-2 + 5}$$
$$\boldsymbol{m = \frac{-7}{3}}$$

## EXERCISE 8G

**1.** Calculate the gradient of the lines connecting the following pairs of points.

(a) (2,4) and (8,10)
(b) (0,3) and (4,11)
(c) (1,3) and (8,17)
(d) (0,0) and (5,8)
(e) (2,3) and (7,9)
(f) (3,4) and (10,8)
(g) (-1,2) and (4,5)
(h) (-2,-3) and (5,0)
(i) (0,4) and (6,0)
(j) (1,8) and (7,-4)
(k) (-4,5) and (3,6)
(l) (-2,-5) and (6,4)
(m) (0,8) and (7,-4)
(n) (2,2) and (12,-7)
(o) (-2,8) and (5,3)
(p) (-3,-1) and (7,-5)
(q) (-4,0) and (0,-8)
(r) (-1,6) and (0,0)
(s) (2,5) and (7,5)
(t) (-3,-4) and (5,-4)

**2.** (a) Which axis ($x$ or $y$) is 1(s) and 1(t) parallel to?
(b) What is the gradient of a line that is parallel to the $x$-axis?

**3.** On the graph shown here, line **A** has a gradient of 1.

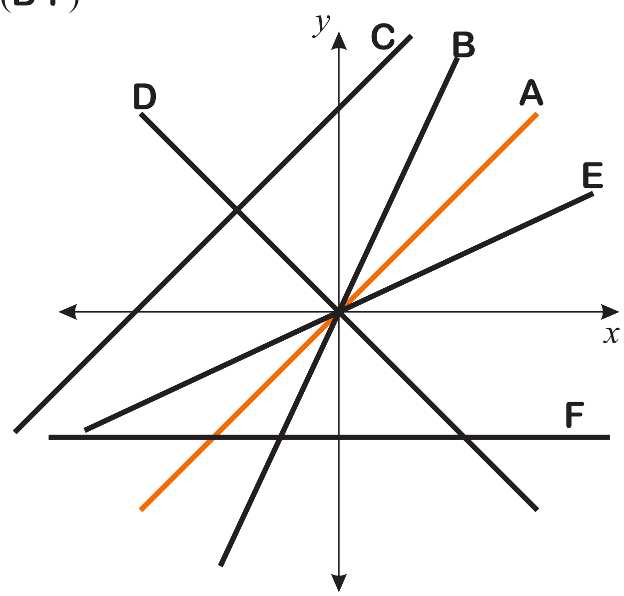
(a) Which of the other lines (**B**-**F**) has a gradient of $\frac{1}{2}$?

(b) Which of the other lines has a gradient of -1?

(c) Which of the other lines has a gradient of 2?

(d) Which of the other lines is parallel to line **A**?

(e) Which of the other lines has a gradient of 0?



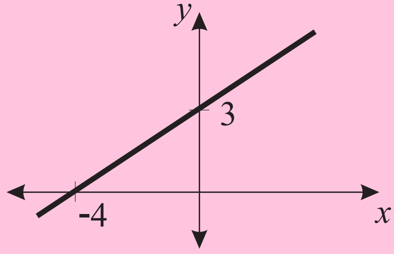
# $x$- and $y$- Intercepts

***Definitions***

The $x$-intercept is the point where a line crosses the $x$-axis.
The $y$-intercept is the point where a line crosses the $y$-axis.

***Examples***

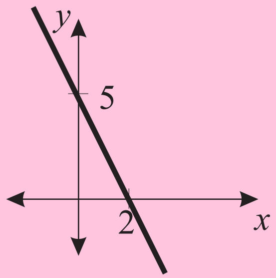
**1.** For the straight line shown on this graph:
the $x$-intercept = -4
the $y$-intercept = 3




The coordinates of the $x$-intercept are (-4,0)
The coordinates of the $y$-intercept are (0,3)

**2.** (a) Sketch the straight line that has the following intercepts:
$x$-intercept = 2
$y$-intercept = 5
(b) State the coordinates of the intercept points.

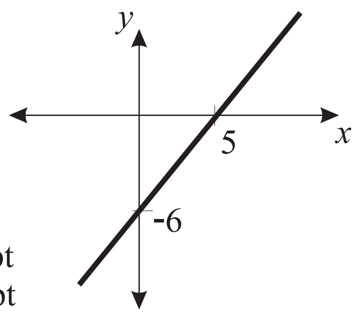
(a)




(b) $x$-intercept = (2,0)
$y$-intercept = (0,5)

## EXERCISE 8H

**1.** For the straight line shown on this graph:
(a) state the $x$-intercept
(b) state the $y$-intercept
(c) state the coordinates of the $x$-intercept
(d) state the coordinates of the $y$-intercept



**2.** Sketch the straight lines that have the following $x$- and $y$-intercepts.
(a) $x$-intercept = 3, $y$-intercept = 2
(b) $x$-intercept = -4, $y$-intercept = 6
(c) $x$-intercept = 5, $y$-intercept = -1
(d) $x$-intercept = -3, $y$-intercept = -5
(e) $x$-intercept = 1, $y$-intercept = -6
(f) $x$-intercept = -7, $y$-intercept = -2

**3.** State the coordinates of the intercept points in question 2.

**4.** Sketch the straight lines that have the following coordinates for their intercept points.
(a) (4,0) and (0,-3)
(b) (-5,0) and (0,5)
(c) (-1,0) and (0,7)
(d) (6,0) and (0,2)

When the equation for a straight line is known, the intercept points can be found by using the following technique.
The straight line can then be sketched.

***Note*** At the $x$-intercept $y = 0$
At the $y$-intercept $x = 0$

***Step 1*** Find the $x$-intercept by substituting $y = 0$ into the equation
***Step 2*** Find the $y$-intercept by substituting $x = 0$ into the equation

***Example 1***

Find the $x$-intercept and $y$-intercept of the straight line with the following equation and sketch the straight line.

$$3x + 2y = 6$$

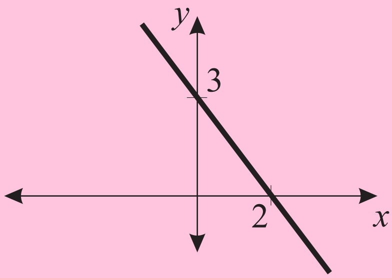
***Step 1*** Find $x$-intercept by substituting $y = 0$ into the equation

$$3x + 2 \times 0 = 6$$
$$3x = 6$$
$$\boldsymbol{x = 2}$$

***Step 2*** Find $y$-intercept by substituting $x = 0$ into the equation

$$3 \times 0 + 2y = 6$$
$$2y = 6$$
$$\mathbf{y = 3}$$

The straight line can now be sketched.




***Example 2***

Find the $x$-intercept and $y$-intercept of the straight line with the following equation and sketch the straight line.

$$y = 2x + 8$$

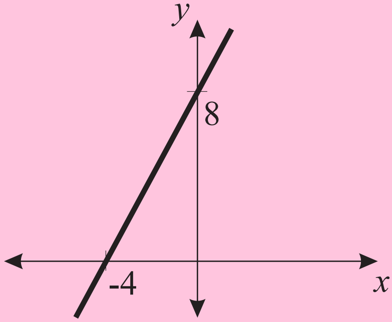
***Step 1*** Find $x$-intercept by substituting $y = 0$ into the equation

$$0 = 2x + 8$$
$$2x + 8 = 0$$
$$2x = -8$$
$$\mathbf{x = -4}$$

***Step 2*** Find $y$-intercept by substituting $x = 0$ into the equation

$$y = 2 \times 0 + 8$$
$$\mathbf{y = 8}$$

The straight line can now be sketched.



**5.** Find the $x$-intercept and $y$-intercept of the straight lines with the following equations and sketch the lines.

(a) $x + y = 3$
(b) $2x + y = 4$
(c) $3x + 3y = 6$
(d) $2x + 5y = 10$
(e) $3x + 8y = 24$
(f) $x + 5y = 10$
(g) $4x + 2y = 12$
(h) $3x + y = 9$
(i) $x - y = 2$
(j) $x - 2y = 4$
(k) $3x - 4y = 12$
(l) $4x - 3y = 24$
(m) $x + 2y = 1$
(n) $2x - y = 1$

**6.** Find the $x$-intercept and $y$-intercept of the straight lines with the following equations and sketch the lines.

(a) $y = 3x - 6$
(b) $y = 2x - 4$
(c) $y = x - 3$
(d) $y = x + 4$
(e) $y = x - 1$
(f) $y = 2x + 6$
(g) $y = 3x - 12$
(h) $y = 4x + 8$
(i) $y = 2x - 10$
(j) $y = 10 - 5x$
(k) $y = 12 - 6x$
(l) $y = 8 - 8x$
(m) $y = 4x + 2$
(n) $y = 6x - 3$

## $y = mx + c$

Equations of straight lines are often written in the form:

$$y = mx + c$$

where ***m = the gradient*** and ***c = the y-intercept***

***Examples***

| *Equation* | *Gradient* | *y-intercept* |
|---|---|---|
| $y = 2x + 5$ | 2 | 5 |
| $y = -3x + 1$ | -3 | 1 |
| $y = \frac{2}{3}x - 4$ | $\frac{2}{3}$ | -4 |
| $y = x$ | 1 | 0 |
| $y = 7 - 4x$ | -4 | 7 |

## EXERCISE 8I

**1.** State the gradient and $y$-intercept of the straight lines with the following equations.

(a) $y = 3x + 4$
(b) $y = 2x + 6$
(c) $y = x + 5$
(d) $y = x - 1$
(e) $y = 2x - 5$
(f) $y = 6x - 7$
(g) $y = -3x + 2$
(h) $y = -4x - 1$
(i) $y = -x - 2$
(j) $y = -5x$
(k) $y = 2x + \frac{1}{2}$
(l) $y = -7x - \frac{3}{5}$
(m) $y = \frac{1}{3}x + 2$
(n) $y = -\frac{3}{4}x - \frac{1}{4}$
(o) $y = 5 + 2x$
(p) $y = 3 - x$
(q) $y = 6 + 3x$
(r) $y = 2 + 8x$
(s) $y = \frac{x}{3} + 1$
(t) $y = \frac{3x}{5} - 4$

**2.** Write the equations of the straight lines with the gradients and $y$-intercepts shown is this table.

| | Gradient | y-intercept |
|---|---|---|
| (a) | 5 | 3 |
| (b) | 2 | -1 |
| (c) | -4 | 5 |
| (d) | -2 | -7 |
| (e) | 1 | -2 |
| (f) | 3 | 0 |
| (g) | $\frac{1}{3}$ | -2 |
| (h) | 2 | $-\frac{2}{9}$ |

**3.** Rearrange the letters of the phrases in capital letters to make this sentence make sense!

Straight lines that are ***ALL PEARL*** have the same ***RED GIANT***.

**4.** State which of the following straight lines are parallel to the line with equation: $y = 3x - 4$.

**A** $y = 4x - 3$
**B** $y = 3x + 4$
**C** $y = x - 4$
**D** $y = x + 3$
**E** $y = 3x - 1$
**F** $y = 3x$
**G** $y = 4x$
**H** $y = 2 - 3x$
**I** $y = 2 + 3x$
**J** $y = 3x + 7$

**5.** Transpose the following equations to make $y$ the subject and hence find the gradient and $y$-intercept.

(a) $y - 3x - 5 = 0$
(b) $y + 2x + 6 = 0$
(c) $y - x + 3 = 0$
(d) $y + x - 2 = 0$
(e) $2y - 6x - 8 = 0$
(f) $2y - 2x + 4 = 0$
(g) $3x + 3y - 9 = 0$
(h) $-8x + 4y - 12 = 0$
(i) $-5 + 2y + 8x = 0$
(j) $12 - 5x + 3y = 0$
(k) $5y - 1 - x = 0$
(l) $3y + 4x = 0$
(m) $6y + 15 - 12x = 0$
(n) $9y + 12 - 6x = 0$

***Example***

$3y - 2x + 6 = 0$

Subtract 6 from both sides

$3y - 2x + 6 = 0$

$\quad\quad\quad - 6 \quad - 6$

$3y - 2x = -6$

Add $2x$ to both sides

$3y - 2x = -6$

$\quad\quad + 2x \quad + 2x$

$3y = 2x - 6$

Divide all terms by 3

$3y = 2x - 6$

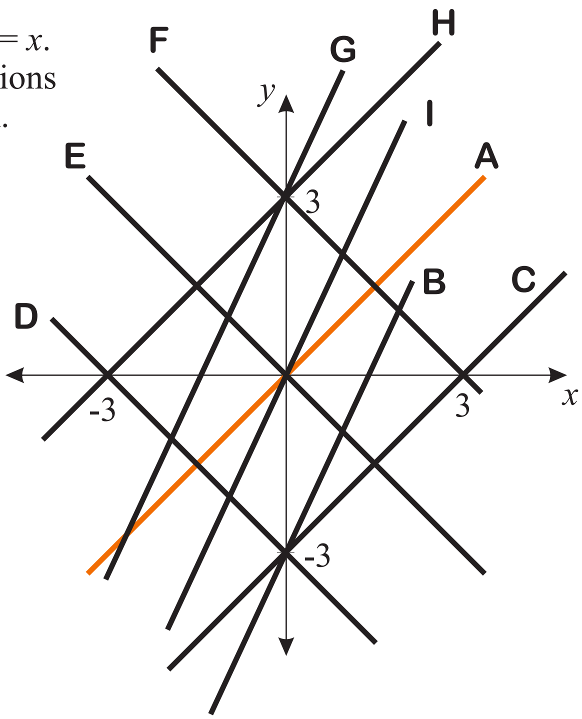
$\div 3 \quad \div 3 \quad \div 3$

$$y = \frac{2}{3}x - 2$$

***Gradient*** $= \frac{2}{3}$, ***y-intercept*** $= -2$

**6.** On the graph shown here, the equation of line **A** is $y = x$. Match the following equations with the lines on the graph.

(a) $y = 2x$
(b) $y = 2x + 3$
(c) $y = x + 3$
(d) $y = -x$
(e) $y = 2x - 3$
(f) $y = -x - 3$
(g) $y = x - 3$
(h) $y = -x + 3$



# Finding the Equation to a Straight Line Given Two Points

***Example***

Find the equation of the straight line that passes through the points (-2,-11) and (4,7) and draw the graph showing these points and the *y*-intercept.

***Step 1*** Find the gradient of the line connecting these two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - -11}{4 - -2}$$

$$= \frac{18}{6}$$

$$m = 3$$

***Step 2*** The value for *m* can be substituted into the equation for a straight line.

$$y = mx + c$$

$$\mathbf{y = 3x + c}$$

***Step 3*** Substitute the *x* and *y* values from either of the points into this equation and solve for *c*.

$$y = 3x + c$$

Use the values from the point (-2,-11).

$$-11 = 3 \times -2 + \text{c}$$

$$-11 = -6 + \text{c}$$

$$+6 \quad +6$$

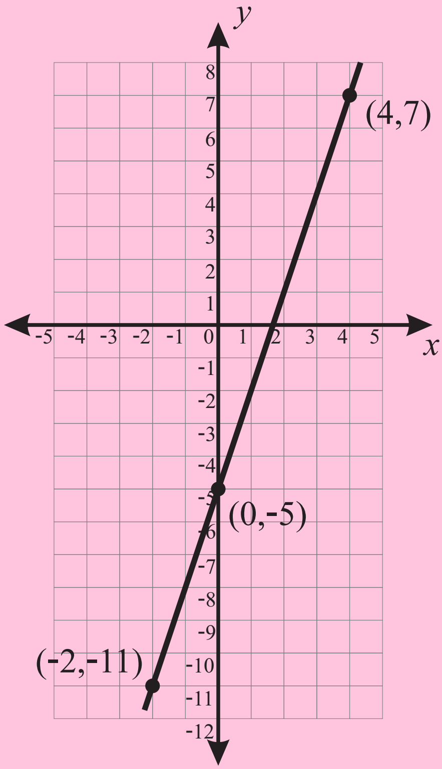
$$-5 = c$$

$$\mathbf{c = -5}$$

***Step 4*** Complete the equation.

$$\mathbf{y = 3x - 5}$$

***Step 5*** Use these three points to draw the graph.
(-2,-11), (4,7) and the *y*-intercept (0,-5)
Ensure the axes are large enough to include all these points.




## EXERCISE 8J

Find the equation of the line that passes through each of the following pairs of points.
Construct axes large enough to draw each line and showing the two points given and the *y*-intercept.

**1.** (-3,-9) and (2,11)
**2.** (-4,-12) and (3,2)
**3.** (-1,7) and (5,-5)
**4.** (-3,7) and (4,-14)
**5.** (-3,-1) and (5,-9)
**6.** (-2,-16) and (4,14)

# Linear Relationships

## EXERCISE 8K

**1.** (a) If $y = 2x$, find $y$ if $x = 3$
(b) If $a = 4b + 1$, find $a$ if $b = 4$
(c) If $M = 3N - 5$, find $M$ if $N = 6$
(d) If $C = 2D - 8$, find $C$ if $D = -1$

**2.** Copy and complete the following tables for the given rules.

***Examples***

**1.** $A = 3B$

| $B$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $A$ | | | | | | | |

***Answer***

| $B$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $A$ | -9 | -6 | -3 | 0 | 3 | 6 | 9 |

**2.** $y = 2x - 3$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

***Answer***

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -9 | -7 | -5 | -3 | -1 | 1 | 3 |

(a) $P = 2Q$

| $Q$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $P$ | | | | | | | |

(b) $n = 3m + 2$

| $m$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $n$ | | | | | | | |

(c) $b = a - 3$

| $a$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $b$ | | | | | | | |

(d) $y = 2x + 5$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(e) $d = 3c - 4$

| $c$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $d$ | | | | | | | |

(f) $M = 4N + 3$

| $N$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $M$ | | | | | | | |

**3.** Copy and complete this table for each of the rules below.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(a) $y = x$
(b) $y = 2x + 3$
(c) $y = x - 5$
(d) $y = 3x - 1$
(e) $y = 4x + 5$
(f) $y = -x$
(g) $y = 2 - x$
(h) $y = 3 - 2x$
(i) $y = 1 - 3x$

**4.** If a graph was drawn for each of the equations in question 3 state the gradient ($m$) and $y$-intercept ($c$) of each graph.

**5.** For each of the tables of values below, find the rule connecting the variables.

***Examples***

**1.**

| $B$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $A$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |

***Answer:*** $A = B + 1$

**2.**

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -5 | -3 | -1 | 1 | 3 | 5 | 7 |

***Answer:*** $y = 2x + 1$

***Hint:*** The $y$ values increase by the coefficient of $x$

(a)

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -12 | -8 | -4 | 0 | 4 | 8 | 12 |

(b)

| $B$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $A$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

(c)

| $c$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $d$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 |

(d)

| $M$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $N$ | -7 | -5 | -3 | -1 | 1 | 3 | 5 |

(e)

| $a$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $b$ | -8 | -5 | -2 | 1 | 4 | 7 | 10 |

(f)

| $P$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $Q$ | -4 | -2 | 0 | 2 | 4 | 6 | 8 |

(g)

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -13 | -9 | -5 | -1 | 3 | 7 | 11 |

(h)

| $m$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $n$ | -10 | -8 | -6 | -4 | -2 | 0 | 2 |

(i)

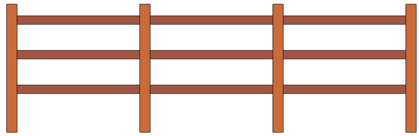
| $C$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $D$ | -11 | -8 | -5 | -2 | 1 | 4 | 7 |

(j)

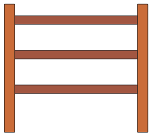
| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -13 | -8 | -3 | 2 | 7 | 12 | 17 |

**6.** A farmer builds a post and rail fence as shown here.

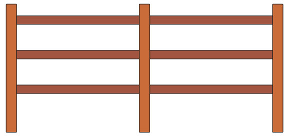
(a) Copy and complete the table below showing the number of rails ($R$) needed for a given number of posts ($P$).



| $P$ | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $R$ | 3 | 6 | | | | |



$P = 2$
$R = 3$



$P = 3$
$R = 6$

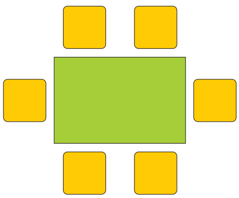
(b) Find the rule connecting $P$ and $R$: $R =$ ____.

(c) Use this rule to find the number of rails needed for the following number of posts:

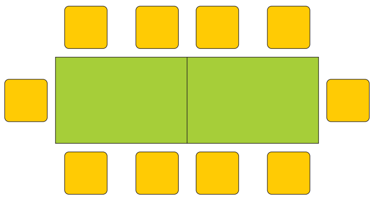
(i) 10 (ii) 25 (iii) 80

**7.** Organisers at a reception centre are wanting to calculate how many chairs are needed to be placed around various numbers of tables. Any number of tables can be joined end to end as shown below.

***Examples***



1 table
6 chairs



2 tables
10 chairs

(a) Copy and complete this table showing the number of chairs ($C$) needed for a given number of tables ($T$).

| $T$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $C$ | 6 | 10 | | |

(b) Find a rule connecting $T$ and $C$:

$C =$ ____.

(c) Use this rule to find the number of chairs needed for the following number of tables:

(i) 6 (ii) 10 (iii) 12

**8.** A cyclist rides at a speed of 6 metres per second.

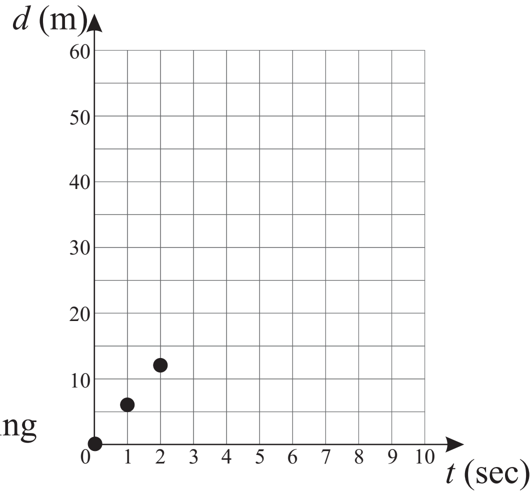
(a) Copy and complete this table showing the distance travelled ($d$) after $t$ seconds.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $d$ | 0 | 6 | 12 | | | | |

(b) Copy and complete this graph showing distance travelled versus time for the first 10 seconds.

(c) Connect the points on the graph with a line.

(d) What shape is the line?




(e) Complete the rule connecting distance ($d$) and time ($t$):

$d =$ ____.

(f) Use this rule to find the distance travelled in the following times:
(i) 20 seconds (ii) 1 minute (iii) 1 hour

**9.** The cost of purchasing tickets for a concert was \$20 per ticket plus a \$10 booking fee.

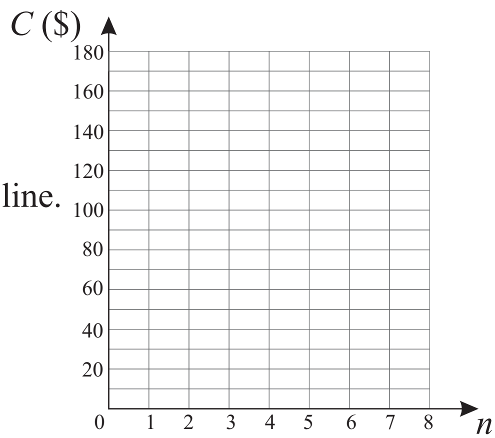
(a) Copy and complete this table showing the cost ($C$) of purchasing a number of tickets ($n$).

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $C$ | 30 | 50 | 70 | | | | | |

(b) Copy and complete this graph showing this information.

(c) Connect the points with a line.

(d) What shape is this line?




(e) Complete this rule connecting cost ($C$) and number of tickets ($n$):

$C =$ ____.

(f) Use this rule to find the cost of purchasing 25 tickets.

**10.** Jim and Tim own rival home gardening services.
Jim charges are \$30 per hour plus \$40 travelling fee.
Tim charges \$40 per hour and no travelling fee.
(a) Copy and complete the following table showing the charge ($C$) for employing each of the gardeners for $n$ hours.

| $n$ hours | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| $C$ - Jim | 70 | 100 | | | | |
| $C$ - Tim | 40 | 80 | | | | |

(b) Display this information on a graph with charge ($C$) on the vertical axis and number of hours ($n$) on the horizontal axis. Show the charges for both gardeners on the one set of axes.
(c) For what number of hours is the charge for each gardener the same?
(d) Find a rule for each of the gardeners that could be used to calculate the charge for a given number of hours.

**11.** A car rental company is offering three different plans for paying for renting a car:

***Plan A*** - \$180 + \$20 per day
***Plan B*** - \$30 + \$40 per day
***Plan C*** - \$50 per day

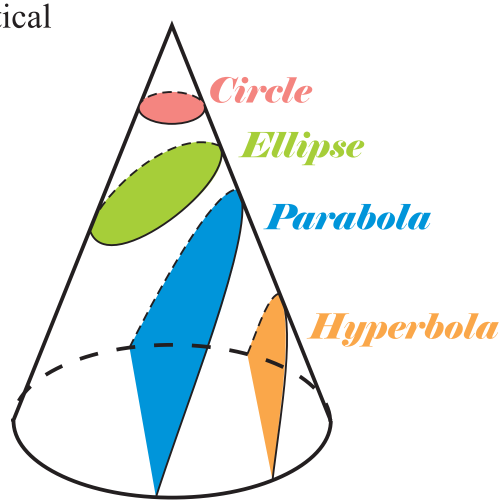
(a) Copy and complete the following table showing the cost ($C$) of renting a car for a number of days ($n$).

| $n$ days | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| $C$ - Plan ***A*** | 200 | | | | | | | |
| $C$ - Plan ***B*** | 70 | | | | | | | |
| $C$ - Plan ***C*** | 50 | | | | | | | |

(b) On the same set of axes draw a graph representing this information.
(c) Which plan would be cheapest for the following number of days?
(i) 2 days (ii) 5 days (iii) 8 days
(d) Find a rule for each of the plans that could be used to calculate the cost for a given number of rental days.

# Non-linear Relationships

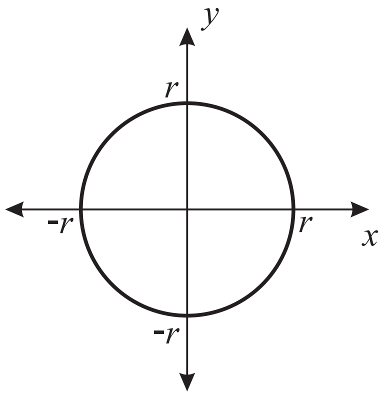
Linear relationships are those that, when graphed, produce straight line graphs. There are many other types of mathematical relationships that produce different shaped graphs.
One family of relationships are ***conics*** or ***conic sections***.
These include ***circles***, ***ellipses***, ***parabolas*** and ***hyperbolas***.
The shapes of these curves can all be obtained by intersecting a cone with a plane as seen on this diagram.
All these graphs are non-linear: not straight lines.




***Circle***

The equation for the basic circle is:

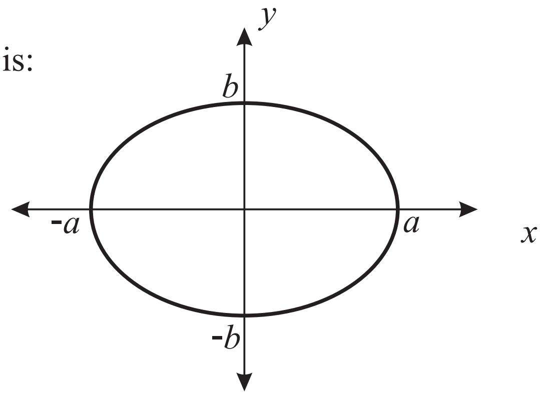
$$x^2 + y^2 = r^2$$




***Ellipse***

The equation for the basic ellipse is:

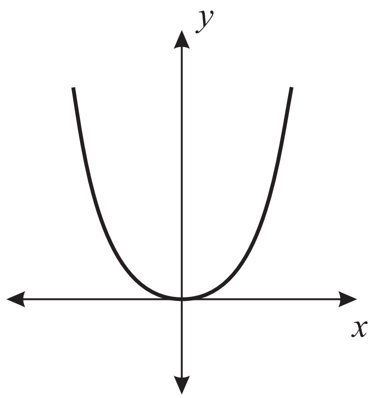
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



***Parabola***

The equation for the basic parabola is:

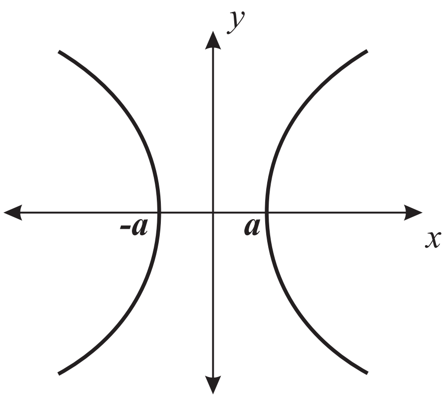
$$y = ax^2$$



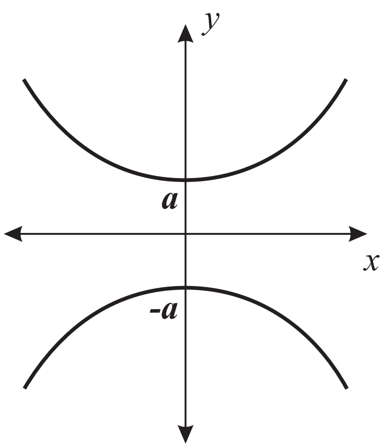

***Hyperbola***

The equation for the two basic hyperbolas are:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$




$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



***Example***

Complete the table of values for the given equation and plot the graph.

$$x^2 + y^2 = 9$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

The equation needs to be transposed to make $y$ the subject.

$$x^2 + y^2 = 9$$

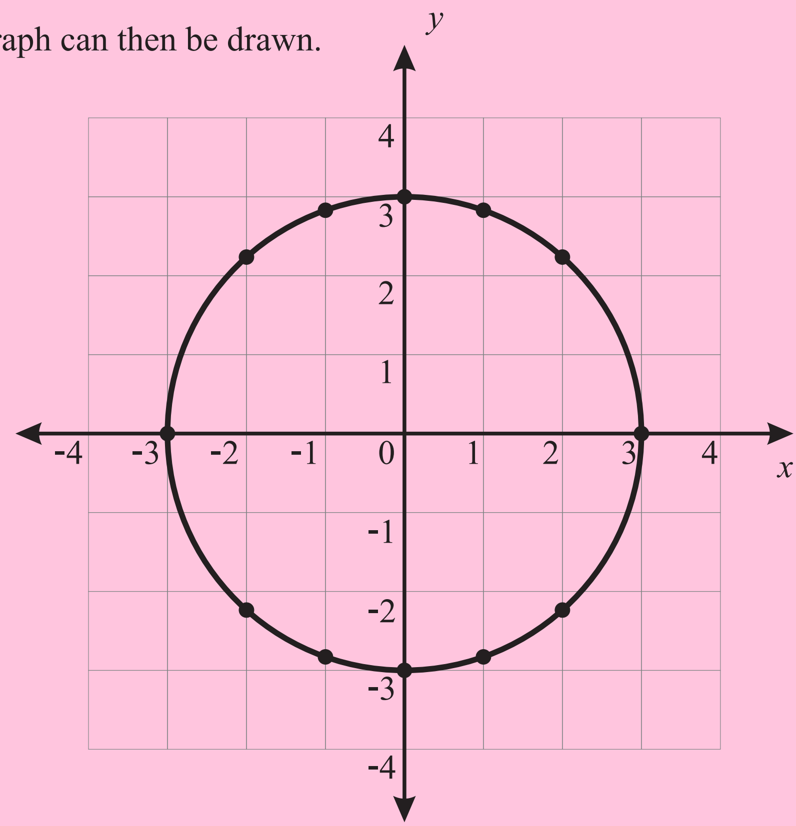
$$y^2 = 9 - x^2$$

$$y = \sqrt{9 - x^2}$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | $\pm\sqrt{5}$ | $\pm\sqrt{8}$ | $\pm 3$ | $\pm\sqrt{8}$ | $\pm\sqrt{5}$ | 0 |

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | ±2.2 | ±2.8 | ±3 | ±2.8 | ±2.2 | 0 |

The graph can then be drawn.



## EXERCISE 8L

**1.** (a) Copy and complete the table of values for the equation below.

$$x^2 + y^2 = 16$$

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

(b) Draw a Cartesian plane with $x$-axis and $y$-axis from -5 to 5.
Plot the points from the table of values and connect with a smooth curve.

**2.** (a) Copy and complete the table of values for the equation below.

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(b) Draw a Cartesian plane with $x$-axis and $y$-axis from -4 to 4.
Plot the points from the table of values and connect with a smooth curve.

**3.** (a) Copy and complete the table of values for the equation below.

$$y = 2x^2$$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(b) Draw a Cartesian plane with $x$-axis from -4 to 4 and $y$-axis from 0 to 20.
Plot the points from the table of values and connect with a smooth curve.

**4.** (a) Copy and complete the table of values for the equation below.

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

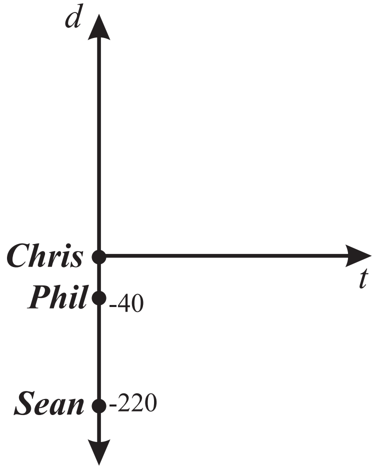
| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(b) Draw a Cartesian plane with $x$-axis from -4 to 4 and $y$-axis from -3 to 3.
Plot the points from the table of values and connect with a smooth curve.

## PROBLEM SOLVING

Chris, Phil and Sean were the only three riders who could win a cycling race. When Chris was 500 metres from the finish line he was 40 metres in front of Phil and 220 metres in front of Sean. Chris was riding at a speed of 10 m/s, Phil was riding at 12 m/s and Sean was riding at 15 m/s. They maintained these speeds until the finish line.

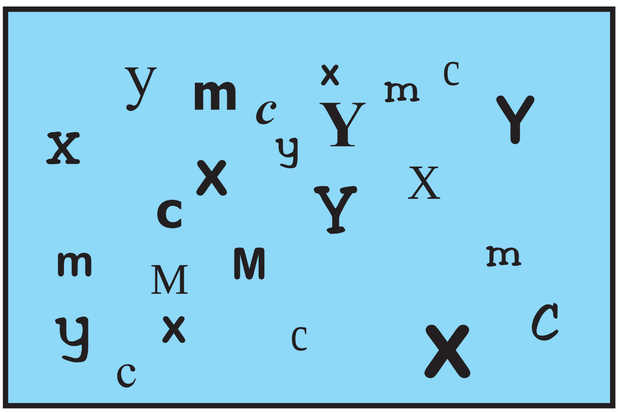
1. On graph paper draw a graph of distance ($d$) versus time ($t$) from the time when Chris is 500 m from the finish line. Draw the distance-time graph for all the three riders on the same axes. The starting points for the graphs are shown here. Continue each line to the point where $d = 500$ (finish line).




2. Who came first, second and third in the race?
3. What was the time between the three riders crossing the finish line?
4. Write an equation for each of these three lines.
5. Find all the points where a rider passed another rider.

## PUZZLE

Use three straight lines to divide this diagram into six sections that each contain a ***y***, ***m***, ***x*** and ***c***.



# CHAPTER REVIEW

**1.** Find the distance between the points (-6,8) and (4,-7). Give answer correct to one decimal place.

**2.** Find the coordinates of the midpoint of the line that connects the points (-8,-7) and (12,-5).

**3.** Find the gradient of the line connecting the following pairs of points.
(a) (0,0) and (2,10) (b) (2,3) and (5,12) (c) (-1,-3) and (1,5)
(d) (-5,4) and (-1,6) (e) (2,1) and (10,-7) (f) (-2,0) and (5,-4)

**4.** Sketch the straight lines with the following $x$- and $y$- intercepts.
(a) $x$-intercept = 2 and $y$-intercept = -4
(b) $x$-intercept = 3 and $y$-intercept = 2

**5.** State the ***coordinates*** of the $x$-intercept and $y$-intercept for the straight lines in question 5.

**6.** State the coordinates of the $x$-intercept and $y$-intercept for the straight lines with the following equations.
(a) $x + y = 5$ (b) $3y - 2x = 6$ (c) $y = x + 1$ (d) $y = 3x - 12$

**7.** State the gradient and $y$-intercept of the straight lines with the following equations.
(a) $y = 3x - 5$ (b) $y = -x + 2$ (c) $y = \frac{2}{3}x - 2$ (d) $y = -4x - \frac{1}{2}$

**8.** Write the equations for the straight lines with the following gradients and $y$-intercepts.
(a) gradient = 2 and $y$-intercept = 6
(b) gradient = -2 and $y$-intercept = -3
(c) gradient = $\frac{3}{4}$ and $y$-intercept = 0

**9.** Transpose the following equations to make $y$ the subject and hence find the $y$-intercept and gradient.
(a) $y + 5x - 3 = 0$ (b) $2y - 3x + 6 = 0$ (c) $4x - 3y + 12 = 0$

**10.** Find the equation of the line that passes through the points (-4,28) and (6,-2).

**11.** Copy and complete the following tables for the given rules.

(a) $y = 3x - 4$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(b) $n = 3 - 2m$

| $m$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $n$ | | | | | | | |

**12.** For each of the tables of values below, find the rule connecting the variables.

(a)

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | -3 | -1 | 1 | 3 | 5 | 7 | 9 |

(b)

| $b$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $a$ | -13 | -10 | -7 | -4 | -1 | 2 | 5 |

**13.** The cost of hiring a jukebox was \$50 plus \$20 per hour.

(a) Write a rule that can be used to calculate the cost ($C$) for a given number of hours ($t$).

(b) How much would it cost to hire the jukebox for 8 hours?